UNIT 5

BEVEL AND WORM GEARS

5.1 BEVEL GEARS

Bevel gears are used to transmit power between shafts whose axes intersect and whose pitch surfaces are rolling cones. Straight toothed bevel gears are the most commonly used bevel gears and it is as shown in Fig. 5.1.

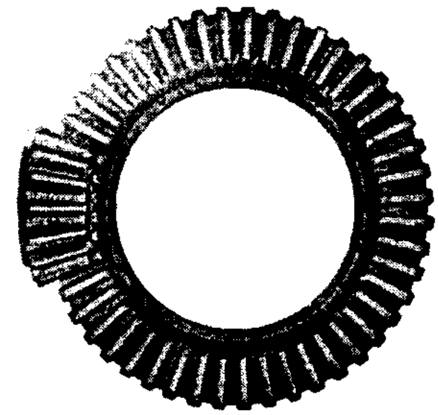


Fig : 5.1

Straight bevel gears impose thrust and radial loads on their support bearings. The teeth are tapered in both thickness and height. The outer part of the tooth or heel is larger than the inner part called the toe. Bevel gears are usually made with unequal addendums in order to avoid interference. The pitch cones must have a common apex, because of this reason bevel gears are designed as a pair and are not interchangeable. The cross section of a bevel gear showing the terminology is shown in Fig:5.2.

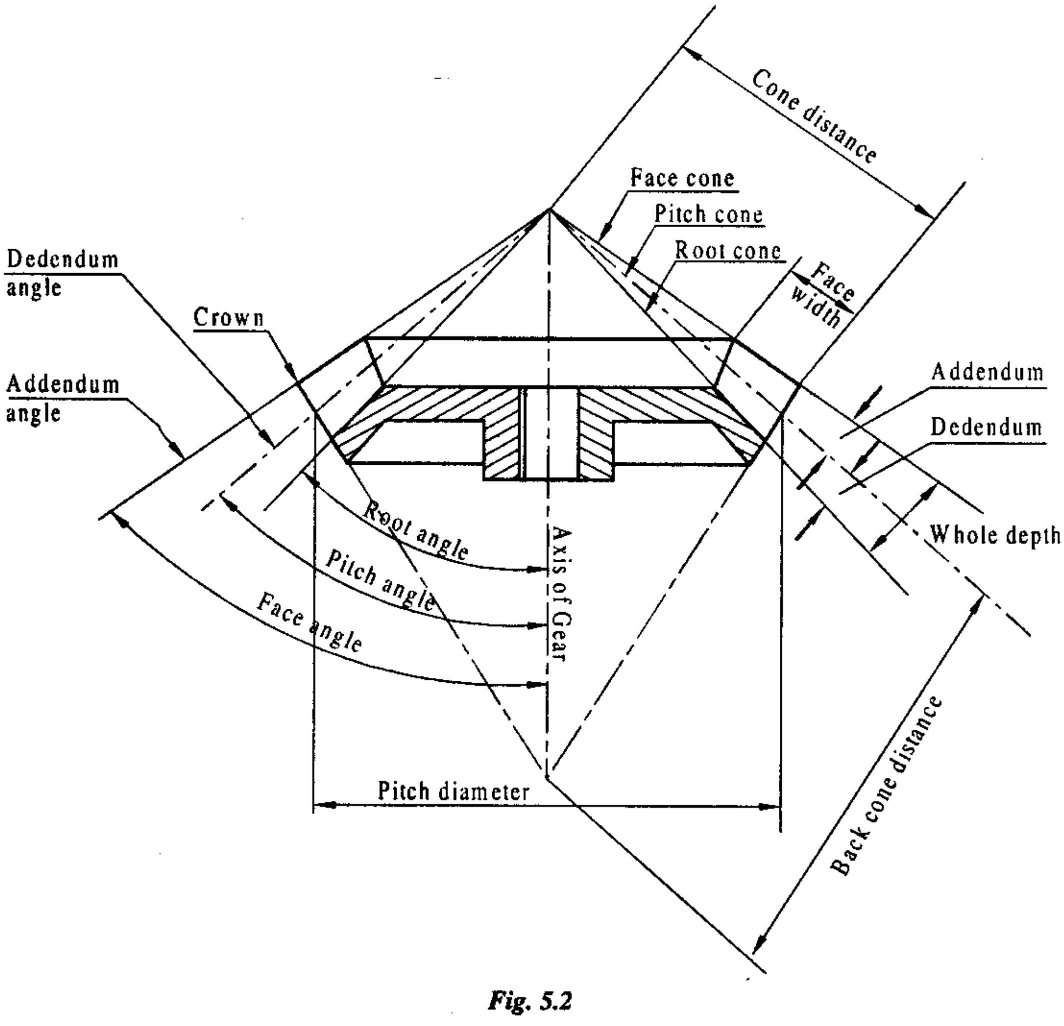


Fig. 5.2

Mitre gear : A pair of bevel gears of the same size and the shafts intersect at right angles are called Mitre gears.

Crown gear : If the pitch angle of the bevel is 90^0 i.e., the pitch cone becomes plane then the gear is called crown gear.

Spiral bevel gear : Spiral bevel gears have curved and oblique teeth. The tooth elements are theoretically spirals, but in practice they are made circular arcs because of the ease of manufacture. Because of the curved teeth, spiral bevel gears have the same advantages over straight-toothed bevel gears as helical gears have over spur gears. Because of the progressive contact, they operate more quietly and have greater tooth strength. A spiral bevel gear is shown in Fig. 5.3.

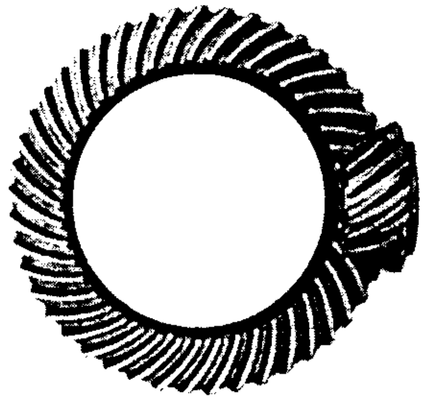


Fig : 5.3

Hypoid gears

Hypoid gears are used to transmit power between nonparallel, non intersecting shafts. They are usually made for shafts which are at 90^0, are made in pairs, and are not interchangeable. In appearance, hypoid gears resemble spiral bevel gears except that the pinion axis is offset above or below the gear axis. The pitch surfaces of hypoid gears are hyper boloids of revolutions. The gear teeth are parallel to the line of contact hence there is sliding along the tooth elements which is a disadvantage. Hypoid gears operate more quietly then spiral bevel gears. Hypoid gears are widely used for automotive rear axle drives because the offset pinion permits lowering the drive shaft and use of a lower body. A hypoid gear is shown in Fig. 5.4.

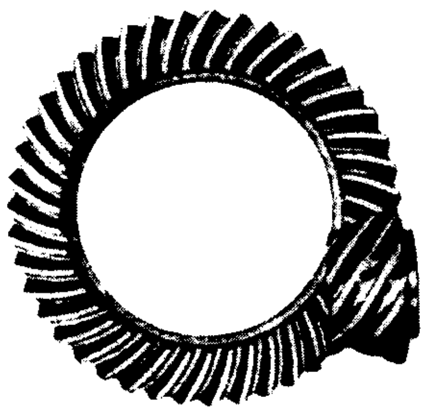


Fig : 5.4

Zerol Bevel gear

Zerol gears are a special form of spiral bevel gear with curved teeth and having a zero degree mean spiral angle. Zerol gear impose the same loads on their support bearings as straight bevel gear.

From Fig. 5.5 (a)

R = Cone distance or slant height of cone

b = face width

δ = half pitch cone angle

δ_1 = half pitch cone angle for pinion

δ_2 = half pitch cone angle for gear

Σ = shaft angle = $\delta_1 + \delta_2$

h_a = Addendum

h_f = Dedendum

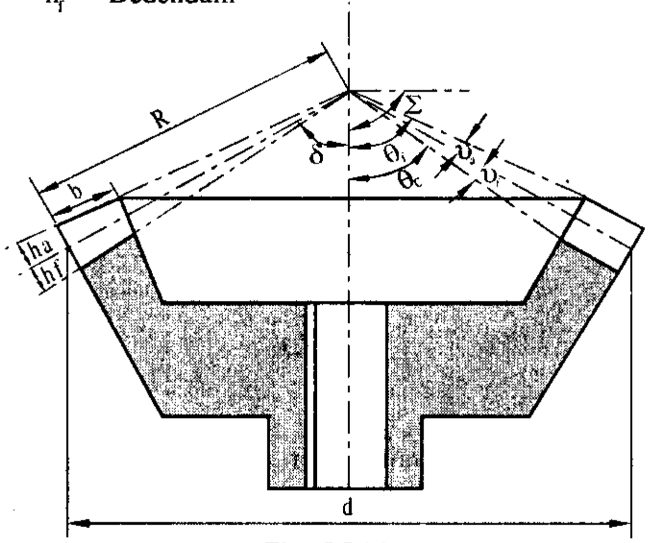


Fig : 5.5 (a)

υ_a = Addendum angle

υ_f = Dedendum angle

θ_c = Cutting angle = $\delta - \upsilon_f$

θ_f = face angle = $\delta + \upsilon_a$

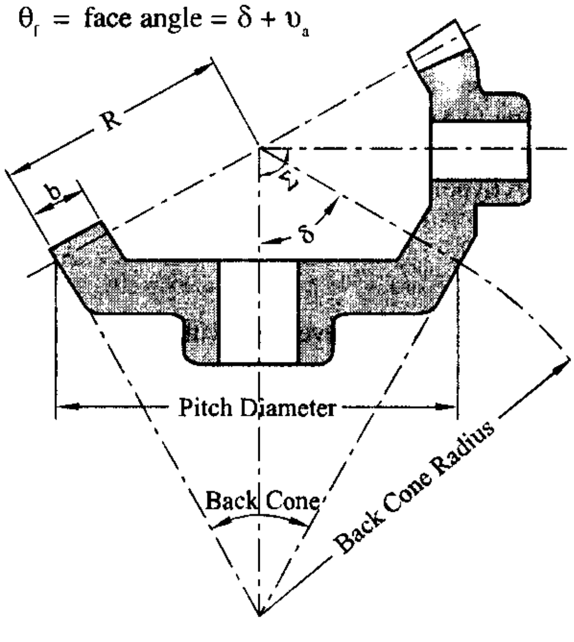


Fig : 5.5 (b)

Acute bevel gear (Fig. 5.6 a)

For acute bevel gear $\Sigma < 90°$

Right angle bevel gear (Fig. 5.6 b)

For right angle bevel gear $\Sigma = 90°$

obtuse bevel gear (Fig. 5.6 c)

For obtuse bevel gear $\Sigma > 90°$

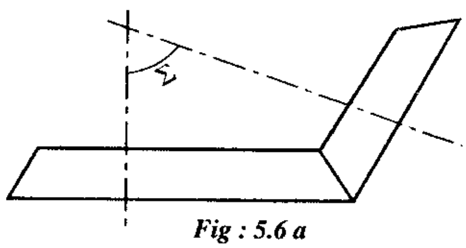


Fig : 5.6 a

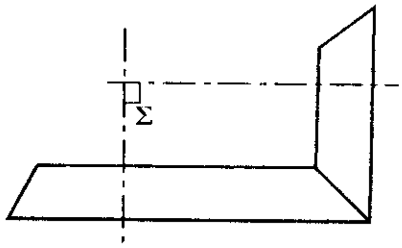


Fig : 5.6 (b)

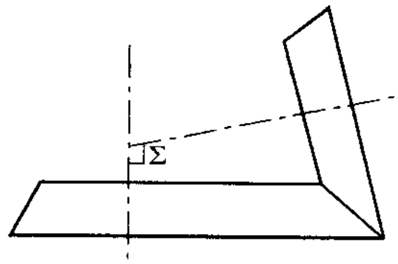


Fig : 5.6 c

5.2 FORMATIVE OR VIRTUAL OR EQUIVALENT NUMBER OF TEETH

The dimensions of a bevel gear are shown in Fig. 5.7. The pitch lines of the teeth lie in the surface of an imaginary cone with the apex at O. The distance R is called the cone distance. The angle d which the pitch line makes with the axis of the gear, is called the pitch angle. The dimensions of the bevel gear are always specified and measured at the large end of the tooth. The addendum h_a the dedendum h_f and the pitch circle diameter d are specified at the large end of the tooth as shown in the Fig. 5.7.

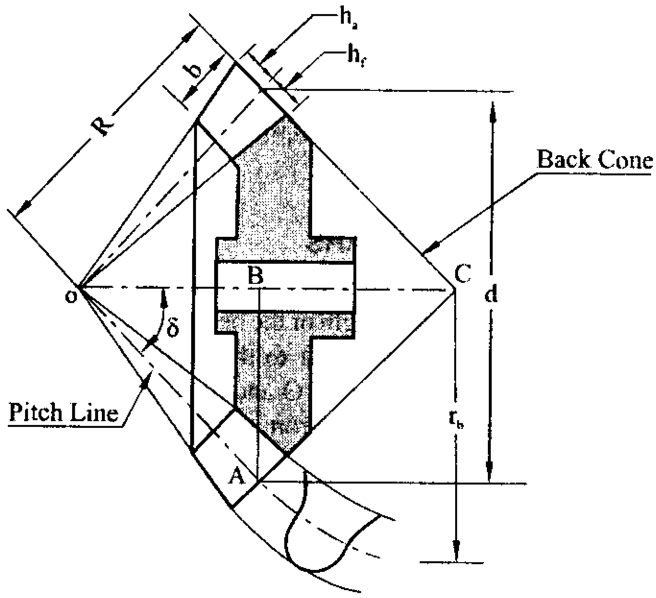


Fig. 5.7

The back cone is an imaginary cone and its elements are perpendicular to the elements of the pitch cone. The radius r_b is called as the back cone radius. For the purpose of design, an imaginary spur gear is considered in a plane perpendicular to the tooth at the large end. r_b is the pitch circle radius of this imaginary spur gear and z_v the number of teeth on this gear. This gear is called the formative gear and the number of teeth z_v on this gear are called virtual or formative number of teeth and are given by

$$z_v = \frac{2r_b}{m} \quad \text{---- (i)}$$

Where m is the module at the large end of the tooth. If z is the actual number of teeth on the bevel gear, then

$$z = \frac{d}{m} \quad \text{---- (ii)}$$

From (i) and (ii) $\frac{z_v}{z} = \frac{2r_b}{d}$ ---- (iii)

From ΔABC

$$\sin \angle BCA = \frac{AB}{AC}$$

$$\text{ie, } \sin(90 - \delta) = \frac{(d/2)}{r_b}$$

$$\therefore \; r_b = \frac{d}{2\cos\delta} \quad \text{---- (iv)}$$

Substituting the above value in equation (iii)

$$z_v = \frac{z}{\cos\delta} = \text{Formative number of teeth on bevel gear} \quad \text{---- (v)}$$

5.3 BEAM STRENGTH OF BEVEL GEARS

The size of the cross-section of the tooth of a bevel gear varies along the face width as shown in Fig. 5.8. In order to determine the beam strength of the bevel gear, it is considered to be equivalent to a formative spur gear in a plane perpendicular to the tooth element. Consider an elemental section of the tooth at a distance x from the apex O and having a width dx. Applying the Lewis equation to a formative spur gear at a distance x from the apex,

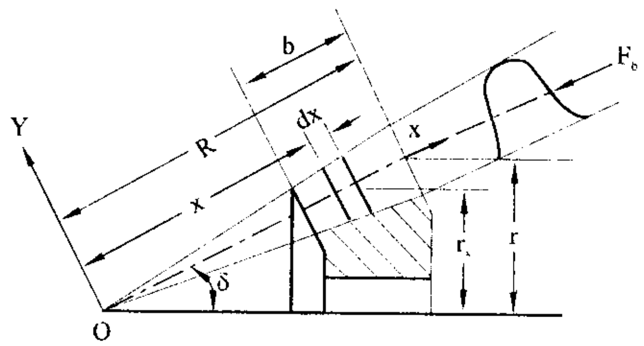


Fig : 5.8

$$\delta(F_b) = m_x b_x \sigma_b Y \quad \text{---- (i)}$$

where $\delta(F_b)$ = beam strength of the elemental section in N

m_x = module for the section in mm

b_x = face width in mm

Y = Lewis form factor based on virtual number of teeth.

From the figure,

$$\frac{r_x}{r} = \frac{x}{R}$$

$$\text{i.e., } r_x = \frac{x.r}{R} \quad \text{---- (ii)}$$

At the element section,

$$m_x = \frac{2r_x}{z} = \frac{2xr}{zR} \quad \text{---- (iii)}$$

At the large end of the tooth,

$$m = \frac{2r}{z} \quad \text{---- (iv)}$$

From (iii) and (iv)

$$m_x = m\left(\frac{x}{R}\right) \quad \text{---- (v)}$$

$$\text{Also } b_x = dx \quad \text{---- (vi)}$$

Substituting (v) and (vi) in (i) we have

$$\delta(F_b) = \frac{m\sigma_b Yx\, dx}{R} \quad \text{---- (vii)}$$

From (ii) and (vii)

$$\int r_x \times \delta(F_b) = \left(\frac{m\sigma_b Yr}{R^2}\right)\int x^2 dx$$

The lefthand side indicates the torque M_t

$$\text{i.e., } M_t = \left(\frac{m\sigma_b Yr}{R^2}\right)\int_{(R-b)}^{R} x^2 dx = \left(\frac{m\sigma_b Yr}{R^2}\right)\left[\frac{x^3}{3}\right]_{(R-b)}^{R}$$

$$\therefore \quad M_t = mb\sigma_b Yr\left[1-\frac{b}{R}+\frac{b^2}{3R^2}\right] \qquad \text{---- (viii)}$$

Assuming beam strength (F_b) as the tangential force at the large end of tooth,

$$M_t = F_b.r \qquad \text{---- (ix)}$$

From (viii) and (ix)

$$F_b = mb\sigma_b Y\left[1-\frac{b}{R}+\frac{b^2}{3R^2}\right] \qquad \text{---- (x)}$$

The face width of the bevel gear is limited to one -third of the cone distance. Therefore, the last in the bracket will never be more than 1/27 and hence neglecting the last term.

$$F_b = mb\sigma_b Y\left[1-\frac{b}{R}\right]$$

where F_b = beam strength of the tooth in N

m = module at the large end of the tooth in mm

b = face width in mm

As per modified form of Lewis equation equivalent tangential force at the larger end of the bevel gear is $F_t = \sigma_0\, C_v\, b\, Y\, m \left(\frac{R-b}{R}\right) = \sigma_0\, C_v\, b\, \pi\, y\, m\left(\frac{R-b}{R}\right)$ ---- 2.426 a

$$\text{where } R = \text{Cone distance} = \frac{1}{2}\sqrt{d_1^2+d_2^2} = \frac{m}{2}\sqrt{z_1^2+z_2^2} \qquad \text{---- 2.414}$$

Width of gear face should lie between $\frac{R}{4} \le b \le \frac{R}{3}$ ---- 2.423

Velocity factor $C_v = \frac{3}{3+v_m}$ for teeth finished with form cutters ---- 2.428

$C_v = \frac{5.55}{5.55+\sqrt{v_m}}$ for the generated system ---- 2.429

The face width of the bevel gear is generally taken as 10m or $\frac{R}{3}$ which ever is smaller.

5.4 DYNAMIC LOAD

In addition to the static load due to power transmission there is a dynamic load between the meshing teeth. According to Buckingham's modified equation.

$$\text{Dynamic load } F_d = F_t + \frac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}} \qquad \text{---- 2.440 a}$$

5.5 WEAR LOAD

The wear strength F_w indicates the maximum value of the tangential force at the large end of the tooth that the tooth can transmit without pitting failure.

$$\text{The limiting wear load } F_w = \frac{d_1 bQK}{\cos\delta_1} \qquad \text{---- 2.441 a}$$

where K = Load stress factor = $\frac{1.43\sigma_{-1c}^2 \sin\alpha}{E_0} = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_0}$

---- 23.161 (New DDHB)

E_0 = Equivalent young's modulus = $\frac{2E_1E_2}{E_1 + E_2}$

$\sigma_{fac} = \sigma_{-1C}$ = Limiting stress for surface fatique = $(2.75\ H_B - 69)$ MPa

---- 23.168 (New DDHB)

Q = Radio factor = $\frac{2z_{v_2}}{z_{v_1} + z_{v_2}}$

b = face width in mm

d_1 = Pitch circle diameter of pinion in mm

δ_1 = half pitch cone angle of pinion

5.6 STRAIGHT TOOTH BEVEL GEAR FORCES

In force analysis, it is assumed that the resultant tooth force between two meshing teeth of bevel gears is concentrated at the midpoint along the face width of the tooth. This radius is given by,

$r_{m1} = \left[\frac{d_1}{2} - \frac{b\sin\delta_1}{2}\right]$ where r_{m1} = mean radius of pinion, b = face width of tooth face, d_1 = pitch diameter of pinion, δ_1 = pitch angle of the pinion. The resultant force has two components F_t and F_s as shown in Fig. 5.9.

F_t = tangential force which is perpendicular to the plane of paper.

F_s = separating force.

Now, $F_t = \frac{M_t}{r_m}$ where M_t = torque transmitted by the gears.

$F_s = F_t \tan\alpha$ where α = Pressure angle.

The separating force is further resolved into two components i.e, axial and radial as shown in Fig . 5.9.

For pinion,

$$F_r = F_s \cos\delta_1 = F_t \tan\alpha \cos\delta_1$$

$$F_a = F_s \sin\delta_1 = F_t \tan\alpha \sin\delta_1$$

where $\tan\delta_1 = \frac{z_1}{z_2}$

$$\therefore \delta_1 = \tan^{-1}\frac{z_1}{z_2} = \tan^{-1}\left(\frac{1}{i}\right) \text{ for right angle bevel gear}$$

---- 2.402 (DDHB)

z_1 = number of teeth on pinion, z_2 = Number of teeth on gear

$$i = \text{Gear ratio} = \left(\frac{z_2}{z_1}\right)$$

The components of the tooth force acting on the gear can be determined by considering actions and reactions as equal and opposite. As seen from the figure 6.17, the radial component of the gear is equal to the axial component on the pinion. Similarly, the axial component on the gear is equal to the radial component on the pinion.

$\therefore$ Axial or thrust on pinion and radial load on gear $F_a = F_t \tan\alpha \sin\delta_1$

Radial load on pinion and Axial or thrust on gear $F_r = F_t \tan\alpha \cos\delta_1$

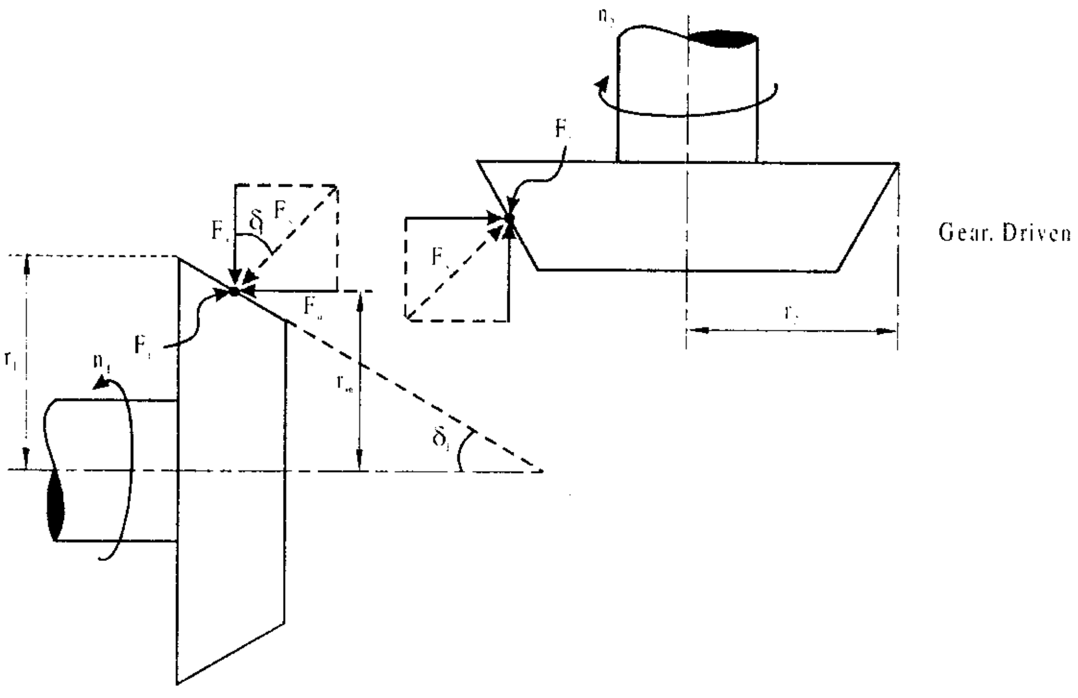


Fig : 5.9

Tangential force $F_t = \frac{M_t}{r_m}$

Mean radius of pinion $r_m = \left[\frac{d_1}{2} - \frac{b\sin\delta_1}{2}\right]$

Pitch angle of pinion $\delta_1 = \tan^{-1}\left(\frac{1}{i}\right)$ for right angle bevel gear.

5.7 PROCEDURAL STEPS FOR THE DESIGN OF BEVEL GEAR

The design procedure for bevel gear is almost the same as that of spur gear

(i) Identity the weaker member

Particulars	σ_o	y	$\sigma_o y$	Remarks
Pinion	σ_{01}	y_1	$\sigma_{01}y_1$	
Gear	σ_{02}	y_2	$\sigma_{02}y_2$	

The gear whose value of $\sigma_o y$ is less is the weaker member i.e., if $\sigma_{01}y_1 < \sigma_{02}y_2$ pinion is weaker

if $\sigma_{02}y_2 < \sigma_{01}y_1$ gear is weaker

Design should be based on weaker member. From Table 2.16 (Old DDHB) ; **Table 23.18 (New DDHB)** obtain the allowable static stresses σ_{01} and σ_{02} for the given materials. For the given pressure angle and tooth form Lewis form factor y for pinion and gear can be obtained using the following formulae.

$$y = 0.124 - \frac{0.684}{z_v} \text{ for } 14\tfrac{1}{2}^\circ \text{ Involute}$$ ---- 2.97(Old) ; **23.115 (New DDHB)**

$$y = 0.154 - \frac{0.912}{z_v} \text{ for } 20^\circ \text{ full depth involute}$$ ---- 2.98(Old) ; **23.116 (New DDHB)**

$$y = 0.17 - \frac{0.95}{z_v} \text{ for } 20^\circ \text{ stub involute}$$ ---- 2.99(Old) ; **23.117 (New DDHB)**

where z_v = Virtual number of teeth = $\frac{z}{\cos\delta}$

Virtual number of teeth on pinion $z_{v_1} = \frac{z_1}{\cos\delta}$ ---- 2.418 (DDHB)

Virtual number of teeth on gear $z_{v_2} = \frac{z_2}{\cos\delta}$ ---- 2.419 (DDHB)

Acute bevel gear

$$\tan\delta_1 = \frac{\sin\Sigma}{\frac{z_2}{z_1} + \cos\Sigma}$$ ---- 2.400 (DDHB)

$$\tan\delta_2 = \frac{\sin\Sigma}{\frac{z_1}{z_2} + \cos\Sigma}$$ ---- 2.401 (DDHB)

For right angle bevel gear

$$\tan\delta_1 = \frac{1}{i}$$ –2.402 (DDHB)

$$\tan\delta_2 = i$$ ---- 2.403 (DDHB)

For obtuse bevel gear

$$\tan\delta_1 = \frac{\sin(180 - \Sigma)}{\frac{z_2}{z_1} - \cos(180 - \Sigma)}$$ ---- 2.404 (DDHB)

$$\tan\delta_2 = \frac{\sin(180-\Sigma)}{\frac{z_1}{z_2} - \cos(180-\Sigma)} \qquad \text{---- 2.405}$$

ii) Design

a) Tangential tooth load $F_t = \dfrac{9550 \times 1000 N C_s}{nr} = \dfrac{9550 \times 1000 P C_s}{nr}$ where r in mm

where r = Pitch circle radius of weaker member in mm = $\dfrac{d}{2}$

C_s = service factor Table 2.33 (Old) ; **Table 23.13 (New DDHB)** or Table 2.86

n = speed of the weaker member

$P = N$ = Power in kW

b) Tangential tooth load from Lewis equation

$$F_t = \sigma_o C_v b Y m \left(\frac{R-b}{R}\right) \qquad \text{---- 2.426 a (DDHB)}$$

where R = cone distance = $\dfrac{1}{2}\sqrt{d_1^2 + d_2^2} = \dfrac{m}{2}\sqrt{z_1^2 + z_2^2}$ ---- 2.414 (DDHB)

For face width $\dfrac{R}{4} < b < \dfrac{R}{3}$

The face width of the bevel gear is generally taken as 10m or $\dfrac{R}{3}$ whichever is smaller.

Lewis form factor $Y = \pi y$

C_v = velocity factor for teeth finished with form cutter = $\dfrac{3}{3 + v_m}$

C_v = velocity factor for generated teeth = $\dfrac{5.55}{5.55 + \sqrt{v_m}}$

By equating the equations obtained from (a) and (b) and by trial and error method find module m.

c) Check for the stress

$$\sigma_{all} = (\sigma_o C_v)_{all}$$

$$\sigma_{ind} = (\sigma_o C_v)_{ind} = \frac{F_t}{bYm\left(\frac{R-b}{R}\right)} \quad \text{---- 2.426 a (DDHB)}$$

If $(\sigma_o C_v)_{ind} < (\sigma_o C_v)_{all}$ than the design is satisfactory.

iii) Dimensions

Calculate all the important geometric parameters of tooth profile by using the equations given in Table 2.1 (Old) ; **Table 23.1 (New DDHB)**

i.e. Addendum h_a, Dedendum h_f, Tooth thickness s,

Total depth h, clearance c, working depth h.

Out side diameter of pinion $d_{a1} = d_1 + 2h_a$

out side diameter of gear $d_{a2} = d_2 + 2h_a$

Dedendum circle diameter of pinion $d_{f1} = d_1 - 2h_f$

Dedendum circle diameter of gear $d_{f2} = d_2 - 2h_f$

$$\text{Addendum angle, } \tan \upsilon_a = \frac{2h_a \sin\delta_1}{d_1} \quad \text{---- 2.406 (DDHB)}$$

$$\text{Dedendum angle, } \tan \upsilon_f = \frac{2h_f \sin\delta_1}{d_1} \quad \text{---- 2.407 (DDHB)}$$

Face angle of pinion, $\theta_{f1} = \delta_1 + \upsilon_a$

Face angle of gear, $\theta_{f2} = \delta_2 + \upsilon_a$

Cutting angle of pinion $\theta_{c1} = \delta_1 - \upsilon_f$

Cutting angle of gear $\theta_{c2} = \delta_2 - \upsilon_f$

If required

Radial load on pinion and thrust load an gear $F_r = F_t \tan\alpha \cos\delta_1$ ----2.456 (DDHB)

Thrust load on pinion and radial load on gear $F_a = F_t \tan\alpha \sin\delta_1$ ---- 2.457 (DDHB)

Neglecting friction

iv) Checking

a) Dynamic load

According to Buckingham's equation

$$\text{Dynamic load } F_d = F_t + \frac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}} \quad \text{---- 2.440 a (DDHB)}$$

To find dynamic load factor C use Tables 2.34 and 2.35 (Old DDHB); Fig. 2.35a and Table 23.32 (New DDHB). Use Fig 2.29 (Old DDHB); Fig. 23.34a (New DDHB) to find the error 'f' if the class of gear is known.

b) Endurance Strength

Endurance strength $F_{-1} = \sigma_{-1}\, by\, \pi m \left(\frac{R-b}{R}\right)$

For safer design F_d must be less than F_{-1}

c) Wear load

According to Buckingham's equation

wear load $F_w = \frac{d_1 bQK}{\cos\delta_1}$ ---- 2.441 a

Ratio factor $Q = \frac{2z_{v_2}}{z_{v_1} + z_{v_2}}$

Load stress factor $K = \frac{1.43\sigma_{-1c}^2 \sin\alpha}{E_0} = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_0}$ ---- 23.161 (New DDHB)

Equivalent young's modulus $E_o = \frac{2\,E_1 E_2}{E_1 + E_2}$

Limiting stress for surface fatigue $\sigma_{fac} = \sigma_{-1c} = (2.75\, H_B - 69)$ MPa

—2.291c(Old); 23.168a (New DDHB)

For safer design F_w must be greater than F_d If $F_w < F_d$ either by increasing the surface hardness or by reducing the error 'f' the required strength can be obtained.

Example 5.1

A pair of bevel gears transmitting 7.5 kW at 300 rpm of pinion. The pressure angle is 20°. The Pitch diameters of pinion and gear at their large ends are 150 mm and 200 mm respectively. The face width of the gears is 40 mm. Determine the components of the resultant gear tooth force and draw a free body diagram of forces acting on the pinion and the gear.

Data :

$$P = N = 7.5\,kW;\ n_1 = 300\,rpm;\ \alpha = 20^0;$$
$$d_1 = 150\,mm;\ d_2 = 200\,mm;\ b = 40\,mm$$

Solution :

$$r_1 = \frac{d_1}{2} = \frac{150}{2} = 75\,mm;\ r_2 = \frac{d_2}{2} = \frac{200}{2} = 100\,mm$$

$$\text{Gear ratio} = i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore\ i = \frac{200}{150} = \frac{4}{3}$$

Torque on the pinion shaft $M_{t_1} = \frac{60\times10^6\times N}{2\pi n_1} = \frac{60\times10^6\times7.5}{2\times\pi\times300} = 238732.4146$ Nmm

Pitch angle of pinion $\delta_1 = \tan^{-1}\frac{1}{i} = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^0$

$\therefore$ Mean radius of Pinion $r_{m_1} = \left[\frac{d_1}{2} - \frac{b\sin\delta_1}{2}\right] = \left[\frac{150}{2} - \frac{40\sin36.87^0}{2}\right] = 63$ mm

$\therefore$ Tangential tooth force $F_t = \frac{M_{t_1}}{r_{m_1}} = \frac{238732.4146}{63} = 3789.4$ N

Axial force on pinion and Radial load on gear $F_a = F_t \tan\alpha\sin\delta_1 = 3789.4\times\tan20\times\sin36.87$
$= 827.54$N

Radial load on pinion and axial force on gear $= F_r = F_t\tan\alpha\cos\delta_1 = 3789.4\times\tan20^0\times\cos36.87$
$= 1103.38$ N

The free body diagram of the forces is shown in Fig. 5.10

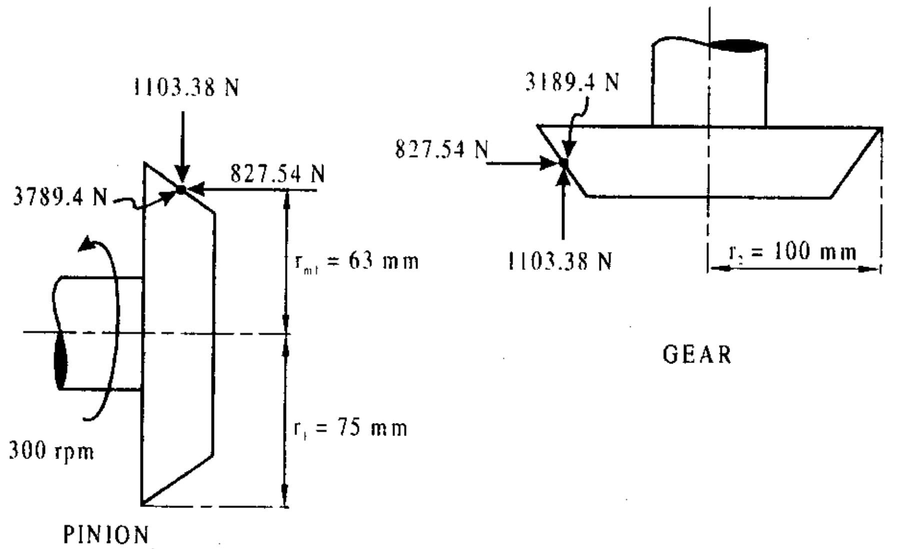


Fig : 5.10

Example 5.2

Design a pair of bevel gears to connect two shafts at 60°. The gears are alloy steel of case hardened and precision cut with form cutters. The gear ratio is 5:1. The power transmitted is 30 kW at 900 rpm of the pinion. The teeth are 20° full depth. The pinion has 24 teeth. Suggest suitable surface hardness for the gear pair.

Data :

$\Sigma = 60°$; $i = 5$; $P = N = 30$ kW; $n_1 = 900$ rpm; $\alpha = 20°$ full depth involute; $z_1 = 24$

Solution :

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{900}{5} = 180\,\text{rpm}$$

Number of teeth on gear $z_2 = i\,z_1 = 5 \times 24 = 120$

Table 23.18 (New DDHB) for alloy steel case hardened (SAE 2320)

$$\sigma_{01} = \sigma_{02} = 345\,\text{MPa} = 345\,\text{N/mm}^2$$

As shaft angle $\Sigma < 90°$, it is acute bevel gear

For acute bevel gear

$$\tan\delta_1 = \frac{\sin\Sigma}{\frac{z_2}{z_1} + \cos\Sigma} = \frac{\sin 60}{\frac{120}{24} + \cos 60} \qquad \text{---2.400}$$

∴ pitch cone angle of pinion $\delta_1 = 8.95°$

$$\tan\delta_2 = \frac{\sin\Sigma}{\frac{z_1}{z_2} + \cos\Sigma} = \frac{\sin 60}{\frac{24}{120} + \cos 60} \qquad \text{----2.401}$$

∴ pitch cone angle of gear $\delta_2 = 51.05°$

Formative number of teeth in a straight tooth bevel pinion

$$z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{24}{\cos 8.95} = 24.296 \qquad \text{----2.418}$$

Formative number of teeth in a straight tooth bevel gear

$$z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{120}{\cos 51.05} = 190.9 \qquad \text{----2.419}$$

Lewis form factor for 20° full depth involute $y = 0.154 - \frac{0.912}{z_v}$ ---- 2.98 (Old DDHB) ; 23.16 (New DDHB)

Form factor for pinion $y_1 = 0.154 - \frac{0.912}{z_{v_1}} = 0.154 - \frac{0.912}{24.296} = 0.11646$

Form factor for gear $y_2 = 0.154 - \frac{0.912}{z_{v_2}} = 0.154 - \frac{0.912}{190.9} = 0.1492$

(i) Identity the weaker member

As pinion and gear are of the same material pinion is the weaker member.

Therefore design should be based an pinion

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000 N C_s}{n.r} = \frac{9550 \times 1000 P C_s}{n.r}$ where r in mm

---- 23.87b (New DDHB)

$\therefore$ Tangential tooth load of the weaker member $F_{t_1} = \frac{9550 \times 1000 N C_s}{n_1 r_1} = \frac{9550 \times 1000 P C_s}{n_1 r_1}$

Pitch circle radius of pinion $r_1 = \frac{d_1}{2} = \frac{m z_1}{2} = \frac{m \times 24}{2} = 12m$

Assume moderate medium shock and 8-10hrs duty per day

$\therefore$ From Table 2.33 (Old DDHB) ; Table 23.13 (New DDHB) or Table 2.86 service factor $C_s = 1.5$

i.e., $F_{t_1} = \frac{9550 \times 1000 \times 30 \times 1.5}{900 \times 12m} = \frac{39791.667}{m}$ ---- (i)

b)Tangential tooth load from Lewis equation $F_t = \sigma_o C_v b Y m \left(\frac{R-b}{R}\right)$ ---- 2.426 a (DDHB)

$\therefore$ Tangential tooth load of the weaker member $F_{t_1} = \sigma_{01} b C_v \pi y_1 m \left(\frac{R-b}{R}\right)$

Cone distance $R = \frac{m}{2}\sqrt{z_1^2 + z_2^2} = \frac{m}{2}\sqrt{24^2 + 120^2} = 61.188m$ ---- 2.414 (DDHB)

For face width $\frac{R}{4} < b < \frac{R}{3}$

$$\frac{R}{3} = \frac{61.188m}{3} = 20.4\,m$$

The face width of the bevel gear is generally taken as 10 m or $\frac{R}{3}$ whichever is smaller or Refer formula 2.425 (DDHB). Since $\frac{R}{3} = 20.4m > 10\,m$, take face width b = 10m

$\therefore F_{t_1} = (345)(10m)(C_v)(\pi \times 0.11646)\,m\left(\frac{61.188m - 10m}{61.188m}\right) = 1055.96\,m^2 C_v$ ---- (ii)

Equating equations (i) and (ii)

$$1055.96\,m^2 C_v = \frac{39791.667}{m}$$

$\therefore m^3 C_v = 37.683$ ---- (iii)

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_1 n_1}{60{,}000}$

$$= \frac{\pi m z_1 n_1}{60{,}000} = \frac{\pi \times m \times 24 \times 900}{60{,}000} = 1.131m$$

Trial : 1 Assume m = 4mm [Select std. module from Table 2.3 (Old DDHB) ; **Table 23.3 (New DDHB)**]

$$\therefore \; v_m = 1.131 \times 4 = 4.524 \text{ m/sec}$$

For teeth finished with form cutters,

$$\text{Velocity factor } C_v = \frac{3}{3+v_m} = \frac{3}{3+4.524} = 0.4$$

From equation (iii)

$$(4^3)0.4 \geq 37.683$$
$$25.6 < 37.683$$

$\therefore$ Not suitable

Trial : 2 Assume m = 5 mm

$$\therefore \; v_m = 1.131 \times 5 = 5.655 \text{ m/sec}$$

$$\therefore \text{ Velocity factor } C_v = \frac{3}{3+5.655} = 0.34662$$

From equation (iii)

$$(5^3)(0.34662) \geq 37.683$$
$$43.327 > 37.683$$

Hence suitable. $\therefore$ Module m = 5 mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{01} C_v)_{all} = (345)(0.34662) = 119.584 \text{ N/mm}^2$

$$\text{Induced stress } \sigma_{ind} = (\sigma_{01} C_v)_{ind} = \frac{F_{t_1}}{b\pi y_1 m\left(\frac{R-b}{R}\right)} \quad \text{---- 2.426a}$$

$$= \frac{\frac{39791.667}{5}}{(10\times5)(\pi\times0.11646)(5)\left(\frac{61.188\times5-10\times5}{61.188\times5}\right)} = 104 \text{ N/mm}^2$$

Since $(\sigma_{01} C_v)_{ind} < (\sigma_{01} C_v)_{all}$ the design is satisfactory. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth i.e., $F_b \geq F_{eff}$

$$F_b = mb\sigma_b Y\left(1-\frac{b}{R}\right)$$

$$= (5)(10\times5)(345)(\pi\times0.11646)\left(1-\frac{10\times5}{61.188\times5}\right)$$

$$= 26399 \text{ N} = \text{Beam strength of the weaker member}$$

$$F_{eff} = \frac{F_t . C_s}{C_v} = \frac{F_t}{C_v} \quad \text{since } C_s \text{ is already considered}$$

$$= \frac{\left(\frac{39791.667}{5}\right)}{0.34662} = 22959.82 \text{ N}$$

As $F_b > F_{eff}$, the design is satis factory from strength point.

iii) Dimensions

From Table 2.1 (Old DDHB) ; Table 23.1 (New DDHB) for $\alpha = 20°$ full depth

Addendum h_a = $1m = 1 \times 5 = 5mm$

Dedendum h_f = $1.25m = 1.25 \times 5 = 6.25$ mm

Working depth h' = $2m = 2 \times 5 = 10mm$

Total depth h = $2.25m = 11.25mm$

Tooth thickness $s = \frac{\pi}{2} m = \frac{\pi}{2} \times 5 = 7.854$ mm

Minimum clearance $c = 0.25\,m = 0.25 \times 5 = 1.25$ mm

Pitch circle diameter of pinion $d_1 = mz_1 = 5 \times 24 = 120$ mm

Pitch circle diameter of gear $d_2 = mz_2 = 5 \times 120 = 600$ mm

Outside diameter of pinion $d_{a1} = d_1 + 2ha_1 = 120 + 2 \times 5 = 130$ mm

Outside diameter of gear $d_{a2} = d_2 + 2ha_2 = 600 + 2 \times 5 = 610mm$

Dedendum circle diameter of pinion $d_{f1} = d_1 - 2h_{f1} = 120 - 2 \times 6.25 = 107.5mm$

Dedendum circle diameter of gear $d_{f2} = d_2 - 2h_{f2} = 600 - 2 \times 6.25 = 587.5mm$

For Addendum angle, $\tan \upsilon_a = \frac{2h_a \sin\delta_1}{d_1} = \frac{2 \times 5 \times \sin 8.95}{120}$ ---- 2.406 (DDHB)

$\therefore$ Addendum angle $\upsilon_a = 0.7428°$

For dedendum angle, $\tan \upsilon_f = \frac{2h_f \sin\delta_1}{d_1} = \frac{2 \times 6.25 \times \sin 8.95}{120}$ ---- 2.407 (DDHB)

$\therefore$ Dedendum angle $\upsilon_f = 0.9284°$

Face angle of pinion $\theta_{f1} = \delta_1 + \upsilon_a = 8.95 + 0.7428 = 9.6928°$

Face angle of gear $\theta_{f2} = \delta_2 + \upsilon_a = 51.05 + 0.7428 = 51.7928$

Cutting angle of pinion $\theta_{c1} = \delta_1 - \upsilon_f = 8.95 - 0.9284 = 8.0216°$

Cutting angle of gear $\theta_{c2} = \delta_2 - \upsilon_f = 51.05 - 0.9284 = 50.1216°$

Tangential tooth load $F_t = \frac{39791.667}{m} = \frac{39791.667}{5} = 7958.33$ N

Face width $b = 10m = 10 \times 5 = 50mm$

Cone distance $R = 61.188\,m = 61.188 \times 5 = 305.94 = 306mm$

Mean pitch line velocity v_m = 5.655m/sec

Velocity factor C_v = 0.34662

Service factor C_s = 1.5

iv) Checking

(a) Dynamic load

According to Buckingham's equation

$$\text{Dynamic load } F_d = F_t + \frac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}} \qquad \text{---- 2.440 a (DDHB)}$$

From Fig 2.29 (Old DDHB) ; **23.34a (New DDHB)** for precision cut and m = 5mm

Error f = 0.0125 mm

From Table 2.35 (Old DDHB) ; **Table 23.32 (New DDHB) for α = 20°** full depth, steel - steel combination and error f = 0.0125 mm

$$\text{Dynamic factor } C = 145\,\frac{\text{kN}}{\text{m}} = 145\ \text{N/mm}$$

$$\text{ie, } F_d = 7958.33 + \frac{21 \times 5.655[7958.33 + 50 \times 145]}{21 \times 5.655 + \sqrt{7958.33 + 50 \times 145}}$$

$$= 15419\text{N}$$

(b) Wear load

According to Buckingham's equation neglecting deflection effect wear load

$$F_w = \frac{d_1 bQK}{\cos\delta_1} \qquad \text{----2.441a}$$

$$\text{Ratio factor } Q = \frac{2z_{v2}}{z_{v1} + z_{v2}} = \frac{2 \times 190.9}{24.296 + 190.9} = 1.7742$$

For safer design $F_w \geq F_d$

$$\text{ie., } \frac{d_1 bQK}{\cos\delta_1} \geq Fd$$

$$\text{ie., } \frac{120 \times 50 \times 1.7742 \times K}{\cos 8.95} \geq 15419$$

$$\therefore K \geq 1.431\ \text{N/mm}^2$$

From Table 23.37B (New DDHB); Table 2.40 (Old DDHB) for α = 20° ; K = 1.431 N/mm²

Surface hardness for pinion = 350 BHN

Surface hardness for gear = 300 BHN

Example 5.3

Design a pair of bevel gears to transmit a power of 25 kW from a shaft rotating at 1200 rpm to a perpendicular shaft to be rotated at 400 rpm.

(VTU, Jan/Feb 2006, Jan/Feb 2005)

Data :

P = N = 25 kW; n_1 = 1200 rpm; n_2 = 400 rpm

Shafts are perpendicular to each other ∴ Σ = 90°

Solution :

Assume

(i) Pressure angle α = 20° full depth involute

(ii) Moderate medium shock and 8 to 10 hrs duty/day

∴ From Table 2.33 (Old DDHB); **Table 23.13 (New DDHB)** or Table 2.86, Service factor C_s = 1.5

(iii) Pinion material as Alloy steel, case hardened (SAE 2320)

∴ Table 23.18 (New DDHB), $\sigma_{01} = 345\ N/mm^2$

(iv) Straight tooth bevel gears

Select the minimum number of teeth on pinion to avoid interference for 20° full depth involute system **(Referring Table 2.92)** $z_1 = 15$

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore \text{ Number of teeth on gear } z_2 = \frac{n_1}{n_2} \cdot z_1 = \frac{1200}{400} \times 15 = 45$$

$$\text{Gear ratio } i = \frac{n_1}{n_2} = \frac{1200}{400} = 3$$

For right angle bevel gear i.e., Σ = 90°

$$\text{Pitch angle of pinion } \delta_1 = \tan^{-1}\left(\frac{1}{i}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.435° \quad \text{---- 2.402 (DDHB)}$$

$$\text{Pitch angle of gear } \delta_2 = \tan^{-1} i = \tan^{-1} 3 = 71.565° \quad \text{---- 2.403 (DDHB)}$$

Formative number of teeth in a straight teeth bevel pinion

$$z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{15}{\cos 18.435} = 15.8114 \quad \text{---- 2.418 (DDHB)}$$

Formative number of teeth in a straight teeth bevel gear

$$z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{45}{\cos 71.565} = 142.3021 \quad \text{---- 2.419 (DDHB)}$$

Lewis form factor for 20° full depth involute system

$$y = 0.154 - \frac{0.912}{z_v} \quad \text{---- 2.98 (Old DDHB); 23.116 (New DDHB)}$$

$$\text{Form factor for pinion } y_1 = 0.154 - \frac{0.912}{z_{v1}} = 0.154 - \frac{0.912}{15.8114} = 0.09632$$

$$\text{Form factor for gear } y_2 = 0.154 - \frac{0.912}{z_{v2}} = 0.154 - \frac{0.912}{142.3021} = 0.1476$$

To select gear material equate $\sigma_{01} y_1$ to $\sigma_{02} y_2$

$$\text{i.e., } 345 \times 0.09632 = \sigma_{02} \times 0.1476; \therefore \sigma_{02} = 225.138 \text{ N/mm}^2$$

From Table 23.18 (New DDHB), select the gear material such that its value of σ_{02} must be nearer to 225.138 N/mm². Hence select steel, SAE 1045, Hardened by WQT as gear material. $\therefore \sigma_{02} = 220$ N/mm²

(i) Identify the weaker member

Particulars	σ_o N/mm²	y	$\sigma_o y$	Remarks
Pinion	345	0.09632	33.23	
Gear	220	0.1476	32.472	weaker

As $\sigma_{02} y_2 < \sigma_{01} y_1$, gear is the weaker member. Therefore design should be based on gear.

(ii) Design

$$\text{(a) Tangential tooth load } F_t = \frac{9550 \times 1000 \times PC_s}{nr} \text{ where r in mm} \quad \text{----- 23.87b (New DDHB)}$$

$$\therefore \text{ Tangential tooth load of the weaker member } F_{t_2} = \frac{9550 \times 1000 NC_s}{n_2 r_2} = \frac{9550 \times 1000 PC_s}{n_2 r_2}$$

$$\text{Pitch circle radius of gear } r_2 = \frac{d_2}{2} = \frac{mz_2}{2} = \frac{m \times 45}{2} = 22.5 \text{ m}$$

$$\therefore F_{t_2} = \frac{9550 \times 1000 \times 25 \times 1.5}{400 \times 22.5m} = \frac{39791.67N}{m} \quad \text{---- (i)}$$

$$\text{(b) Tangential tooth load from Lewis equation } F_t = \sigma_o C_v b Y m \left(\frac{R-b}{R}\right) \quad \text{----- 2.426 a (DDHB)}$$

$$\text{Tangential tooth load of the weaker member } F_{t_2} = \sigma_{o_2} C_v b \pi y_2 m \left(\frac{R-b}{R}\right)$$

$$\text{Cone distance } R = \frac{m}{2}\sqrt{z_1^2 + z_2^2} = \frac{m}{2}\sqrt{15^2 + 45^2} = 23.717 \text{ m} \quad \text{----- 2.414 (DDHB)}$$

For face width $\frac{R}{4} < b < \frac{R}{3}$

$$\frac{R}{3} = \frac{23.717m}{3} = 7.9\,m$$

The face width of the bevel gear is generally taken as 10m or $\frac{R}{3}$ which ever is smaller

As $\frac{R}{3} = 7.9m < 10m$, take face width b = 8m

$$\therefore F_{t_2} = (220)(C_v)(8m)(\pi \times 0.1476)(m)\left(\frac{23.717m - 8m}{23.717m}\right) = 540.83\,m^2 C_v \qquad \text{-----(ii)}$$

Equating the equation (i) and (ii)

$$\text{i.e.,} \quad \frac{39791.67}{m} = 540.83\,m^2 C_v$$

$$\therefore m^3 C_v = 73.576 \qquad \text{-----(iii)}$$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_2 n_2}{60{,}000} = \frac{\pi m z_2 n_2}{60{,}000}$

$$= \frac{\pi \times m \times 45 \times 400}{60{,}000} = 0.9425\,m$$

Trial : 1 Assume m = 5 mm [Select standard module from Table 23.3 (New DDHB); Table 2.3 (Old DDHB)]

$$\therefore v_m = 0.9425 \times 5 = 4.7125\text{ m/sec}$$

Assume the teeth are finished with form cutters.

$$\therefore \text{Velocity factor } C_v = \frac{3}{3 + v_m} = \frac{3}{3 + 4.7125} = 0.389 \qquad \text{-----2.428 (DDHB)}$$

From equation (iii)

$$(5^3)(0.389) \geq 73.576$$

$$48.625 < 73.576$$

∴ Not suitable.

Trial : 2 Assume m = 6 mm

$$v_m = 0.9425 \times 6 = 5.655\text{ m/sec}$$

$$\therefore \text{Velocity factor } C_v = \frac{3}{3 + 5.655} = 0.34662$$

From equation (iii)

$$(6^3)(0.34662) \geq 73.576$$

$$74.87 < 73.576$$

Hence suitable $\therefore$ Module m = 6mm

(c) Check for the stress

$$\text{Allowance stress } \sigma_{all} = (\sigma_{o2} C_v)_{all} = (220)(0.34662) = 76.256 \text{ N/mm}^2$$

$$\text{Induced stress } \sigma_{ind} = (\sigma_{o2} C_v)_{ind} = \frac{F_{t_2}}{b\pi y_2 m\left(\frac{R-b}{R}\right)}$$

$$= \frac{\frac{39791.67}{6}}{(8\times6)(\pi\times0.1476)(6)\left(\frac{23.7170\times6-8\times6}{23.717\times6}\right)}$$

$$= 74.938 \text{ N/mm}^2$$

As $(\sigma_{02}C_v)_{ind} < (\sigma_{02}C_v)_{all}$, the design is satisfactory.

$\therefore$ Module m = 6mm

(iii) Dimensions

From Table 23.1 (New DDHB); Table 2.1 (Old DDHB) for $\alpha = 20°$ Full depth

Addendum $h_a = 1m = 1\times6 = 6$ mm

Dedendum $h_f = 1.25\,m = 1.25\times6 = 7.5$ mm

Working depth $h' = 2\,m = 2\times6 = 12$ mm

Total depth $h = 2.25\;m = 2.25\times6 = 13.5$ mm

$$\text{Tooth thickness } s = \frac{\pi m}{2} = \frac{\pi\times6}{2} = 9.425 \text{ mm}$$

Minimum clearance $c = 0.25\,m = 0.25\times6 = 1.5$ mm

Pitch circle diameter of pinion $d_1 = mZ_1 = 6\times15 = 90$ mm

Pitch circle diameter of gear $d_2 = mZ_2 = 6\times45 = 270$ mm

Outside diameter of pinion $da_1 = d_1 + 2h_{a1} = 90 + 2\times6 = 102$ mm

Outside diameter of gear $da_2 = d_2 + 2h_{a2} = 270 + 2\times6 = 282$ mm

Dedendum circle diameter of pinion $d_{f1} = d_1 - 2h_{f1} = 90 - 2\times7.5 = 75$ mm

Dedendum circle diameter of pinion $d_{f2} = d_2 - 2h_{f2} = 270 - 2\times7.5 = 255$ mm

$$\text{For addendum angle, } \tan \upsilon_a = \frac{2h_a \sin\delta_1}{d_1} = \frac{2\times6\times\sin 18.435}{90}$$

$\therefore$ Addendum angle $\upsilon_a = 2.414°$

For dedendum angle, $\tan \upsilon_f = \dfrac{2h_f \sin\delta_1}{d_1} = \dfrac{2 \times 7.5 \times \sin 18.345}{90}$

$\therefore$ Dedendum angle $\upsilon_f = 3.017°$

Face angle of pinion $\theta_{f1} = \delta_1 + \upsilon_a = 18.435 + 2.414 = 20.849°$

Face angle of gear $\theta_{f2} = \delta_2 + \upsilon_a = 71.565 + 2.414 = 73.979°$

Cutting angle of pinion $\theta_{c1} = \delta_1 - \upsilon_f = 18.435 - 3.017 = 15.418°$

Cutting angle of gear $\theta_{c2} = \delta_2 - \upsilon_f = 71.565 - 3.017 = 68.548°$

Tangential took load $F_t = \dfrac{39791.67}{6} = 6631.945\ N$

Face width $b = 8\,m = 8 \times 6 = 48\ mm$

Cone distance $R = 23.717\,m = 23.717 \times 6 = 142.3\ mm$

Mean pitch line velocity $v_m = 5.655\ m/sec$

Velocity factor $C_v = 0.34662$

Service factor $C_s = 1.5$

(iv) Checking

(a) Dynamic load

According to Buckingham's equation

$$\text{Dynamic load } F_d = F_t + \frac{21 v_m (F_t + bc)}{21 v_m + \sqrt{F_t + bc}} \qquad \text{---- 2.440 a (DDHB)}$$

From Fig. 23.35a (New DDHB) ; Fig. 2.30 (Old DDHB) for $v_m = 5.655\ m/sec$

Error $f = 0.063\ mm$

From Table 23.32 (New DDHB) ; Table 2.35 (Old DDHB) for $\propto = 20°$ Full depth, steel-steel combination

when error $f = 0.05\ mm$, $C = 580\ kN/m = 580\ N/mm$

when error $f = 0.075\ mm$, $C = 870\ kN/m = 870\ N/mm$

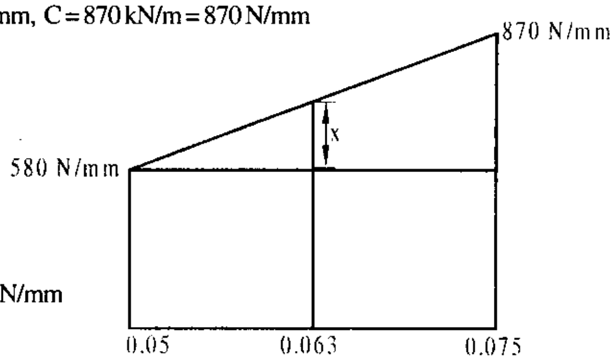
By interpolation

$\dfrac{x}{870 - 580} = \dfrac{0.063 - 0.05}{0.075 - 0.05}$

$x = 150.8\ N/mm$

$\therefore$ When error $f = 0.063\ mm$,

Dynamic factor $C = 580 + 150.8 = 730.8\ N/mm$



$$F_d = 6631.945 + \frac{21\times5.655[6631.945+48\times730.8]}{21\times5.655+\sqrt{6631.945+48\times730.8}} = 21967.936\,N$$

(b) Wear load

According to Buckingham's equation neglecting deflection effect, wear load $F_w = \frac{d_1 bQK}{\cos\delta_1}$

---- 2.441 a (DDHB)

$$\text{Ratio factor } Q = \frac{2Z_{v_2}}{Z_{v_1}+Z_{v_2}} = \frac{2\times142.3021}{15.8114+142.3021} = 1.8$$

For safer design

$$F_w \geq F_d$$

$$\text{ie., } \frac{d_1 bQK}{\cos\delta_1} \geq F_d$$

$$\text{ie., } \frac{90\times48\times1.8\times K}{\cos 18.435} \geq 21967.936$$

$$\therefore K \geq 2.68\,N/mm2$$

From Table 23.37B (New DDHB); Table 2.40 (Old DDHB) for $\alpha = 20°$ and $K \geq 2.68\ N/mm^2$

Surface hardness for pinion = 450 BHN

Surface hardness for gear = 450 BHN

Example 5.4

Design a pair of bevel gears to transmit 12 kW at 300 rpm of the gear and 1470 rpm of the pinion. The angle between the shaft axes is 90°. The pinion has 20 teeth and the material for gears is cast steel (σ_0 = 183.33 N/mm², BHN 320. Take service factor as 1.25 and check the gears for wear and dynamic load. Suggest suitable surface hardness for the gear pair. **VTU, July/August 2002**

Data :

$$P = N = 12\,kW;\ n_2 = 300\,rpm;\ n_1 = 1470\,rpm$$

$$\Sigma = 90°;\ z_1 = 20;\ C_s = 1.25$$

$$\sigma_{01} = \sigma_{02} = 183.33\,N/mm^2;\ BHN = 320$$

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore i = \frac{1470}{300} = 4.9$$

$$\text{Number of teeth on gear } z_2 = iz_1 = 4.9\times20 = 98$$

For right angle bevel gear, i.e., $\Sigma = 90°$

pitch cone angle of pinion $\delta_1 = \tan^{-1}\left(\frac{1}{i}\right) = \tan^{-1}\left(\frac{1}{4.9}\right) = 11.535°$ ----2.402 (DDHB)

Pitch cone angle of gear $\delta_2 = \tan^{-1} i = \tan^{-1} 4.9 = 78.465°$ ----2.403 (DDHB)

Formative number of teeth in a straight tooth bevel gear $z_v = \frac{z}{\cos\delta}$

$\therefore$ Formative number of teeth on pinion $z_{v_1} = \frac{z_1}{\cos\delta_1} = \frac{20}{\cos 11.935} = 20.442$

Formative number of teeth on gear $z_{v_2} = \frac{z_2}{\cos\delta_2} = \frac{98}{\cos 78.465} = 490$ ----2.419 (DDHB)

Assume α = 20° full depth

Lewis form factor for 20° full depth involute $y = 0.154 - \frac{0.912}{z_v}$ ---- 2.98 (Old) ; **23.116 (New DDHB)**

Form factor for pinion $y_1 = 0.154 - \frac{0.912}{z_{v_1}} = 0.154 - \frac{0.912}{20.442} = 0.1094$

Form factor for gear $y_2 = 0.154 - \frac{0.912}{z_{v_2}} = 0.154 - \frac{0.912}{490} = 0.15214$

i) Identify the weaker member

As the pinion and gear are of the same material, pinion is weaker member

Therefore design should be based on pinion.

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000 NC_S}{nr} = \frac{9550 \times 1000 \times PC_S}{nr}$ where r in mm

$\therefore$ Tangential tooth load of the weaker member $F_{t_1} = \frac{9550 \times 1000 NC_s}{n_1 r_1} = \frac{9550 \times 1000 \times PC_s}{n_1 r_1}$

where r_1 = pitch circle radius of pinion $= \frac{d_1}{2} = \frac{mz_1}{2} = \frac{m \times 20}{2} = 10\,m$

$$\therefore F_{t_1} = \frac{9550 \times 1000 \times 12 \times 1.25}{1470 \times 10m} = \frac{9744.9}{m} \quad \text{----(i)}$$

b) Tangential tooth load from Lewis equation $F_t = \sigma_o C_v b Y m \left(\frac{R-b}{R}\right)$ ---- 2.426 a (DDHB)

$\therefore$ Tangential tooth load of the weaker member $F_{t1} = \sigma_{01} b C_v \pi y_1 m \left(\frac{R-b}{R}\right)$

Cone distance $R = \frac{m}{2}\sqrt{z_1^2 + z_2^2} = \frac{m}{2}\sqrt{20^2 + 98^2} = 50\,m$ ----2.414 (DDHB)

For face width $\frac{R}{4} < b < \frac{R}{3}$

$$\frac{R}{3} = \frac{50}{3} = 16.67\,m$$

The face width of the bevel gear is generally taken as 10 m or $\frac{R}{3}$ whichever is small

or Refer formula 2.425 (DDHB)

$$\text{Since } \frac{R}{3} = 16.67\,m > 10\,m$$

$$\text{Take face width } b = 10\,m$$

$$\therefore F_{t1} = (183.33)(C_v)(10\,m)(\pi \times 0.1094)\,m\left(\frac{50m - 10m}{50m}\right) = 504.07\,m^2 C_v \qquad \text{----(ii)}$$

Equating equations (i) and (ii)

$$504.07\,m^2 C_v = \frac{9744.9}{m}$$

$$\text{i.e., } m^3 C_v = 19.332 \qquad \text{----(iii)}$$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_1 n_1}{60000}$

$$= \frac{\pi m z_1 n_1}{60000} = \frac{\pi \times m \times 20 \times 1470}{60000} = 1.5394\,m$$

Trial : 1

Select module m= 3 mm [Select standard module from Table 2.3 (Old DDHB); **Table 23.3 (New DDHB)]**

$$\therefore v_m = 1.5394 \times 3 = 4.618\ \text{m/sec}$$

Assume the teeth are generated.

$$\therefore \text{Velocity factor } C_v = \frac{5.55}{5.55 + \sqrt{v_m}} = \frac{5.55}{5.55 + \sqrt{4.618}} = 0.7209 \qquad 2.429\,(\text{DDHB})$$

From equation (iii)

$$(3^3)(0.7209) \geq 19.332$$

$$19.46 > 19.332$$

$\therefore$ Suitable. Hence module m = 3 mm

c) Check for the stress

$$\text{Allowable stress } \sigma_{all} = (\sigma_{01} \cdot C_v)_{all} = (183.33)(0.7209) = 132.16\ \text{N/mm}^2$$

$$\text{Induced stress } \sigma_{ind} = (\sigma_{01} C_v)_{ind} = \frac{F_{t1}}{b \pi y_1 m\left(\frac{R-b}{R}\right)} \qquad \text{----2.426 a}$$

$$= \frac{\left(\frac{9744.9}{3}\right)}{(10 \times 3)(\pi \times 0.1094)(3)\left(\frac{50 \times 3 - 10 \times 3}{50 \times 3}\right)} = 132.27\ \text{N/mm}^2$$

Since $(\sigma_{ol}C_v)_{ind} < (\sigma_{ol}C_v)_{all}$, the design is satisfactory. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

$$\text{i.e., } F_b \geq F_{eff}$$

$$F_b = mb\sigma_b Y\left(1-\frac{b}{R}\right)$$

$$= (3)(10\times3)(183.33)(\pi\times0.1094)\left(1-\frac{10\times3}{50\times3}\right) = 4536.63\text{ N}$$

$$F_{eff} = \frac{F_t . C_s}{C_v} = \frac{F_t}{C_v} \text{ since } C_s \text{ is already considered}$$

$$= \frac{\left(\frac{9744.9}{3}\right)}{0.7209} = 4505.9\text{ N}$$

Since $F_b > F_{eff}$, the design is satisfactory from strength point

iii) Dimensioning

From Table 2.1 (Old DDHB) ; **Table 23.1 (New DDHB)** for $\alpha = 20°$ full depth

Addendum	h_a	$= 1\text{ m} = 1\times3 = 3\text{ mm}$
Dedendum	h_f	$= 1.25\text{ m} = 1.25\times3 = 3.75\text{ mm}$
Working depth	h'	$= 2\text{ m} = 2\times3 = 6\text{ mm}$
Total depth	h	$= 2.25\text{ m} = 2.25\times3 = 6.75\text{ mm}$
Tooth thickness	s	$= \frac{\pi}{2}\text{m} = \frac{\pi}{2}\times3 = 4.7124\text{ mm}$
Minimum clearance	c	$= 0.25\text{ m} = 0.25\times3 = 0.75\text{ mm}$
Pitch circle diameter of pinion	d_1	$= mz_1 = 3\times20 = 60\text{ mm}$
Pitch circle diameter of gear	d_2	$= mz_2 = 3\times98 = 294\text{ mm}$
Outside diameter of pinion	da_1	$= d_1 + 2h_a = 60 + 2\times3 = 66\text{ mm}$
Outside diameter of gear	da_2	$= d_2 + 2ha = 294 + 2\times3 = 300\text{ mm}$

Dedendum circle diameter of pinion $df_1 = d_1 - 2h_f = 60 - 2\times3.75 = 52.5\text{ mm}$

Dedendum circle diameter of pinion $df_2 = d_2 - 2h_f = 294 - 2\times3.75 = 286.5\text{ mm}$

Addendum angle $$\upsilon_a = \tan^{-1}\left(\frac{2h_a\sin\delta_1}{d_1}\right) = \tan^{-1}\left(\frac{2\times3\times\sin11.535}{60}\right) = 1.1456°$$

Dedendum angle $$\upsilon_f = \tan^{-1}\left(\frac{2h_f\sin\delta_1}{d_1}\right) = \tan^{-1}\left(\frac{2\times3.75\times\sin11.535}{60}\right) = 1.432°$$

Face angle of pinion	θ_{f1}	$= \delta_1 + \upsilon_a = 11.535 + 1.1456 = 12.6806°$
Face angle of gear	θ_{f2}	$= \delta_2 + \upsilon_a = 78.465 + 1.1456 = 79.6016°$
Cutting angle of pinion	θ_{c1}	$= \delta_1 - \upsilon_f = 11.535 - 1.432 = 10.103°$
Cutting angle of gear	θ_{c2}	$= \delta_2 - \upsilon_f = 78.465 - 1.432 = 77.033°$

Tangential tooth load	F_t	$= \dfrac{9744.9}{3} = 3248.3\,N$
Face width	b	$= 10m = 10 \times 3 = 30\,mm$
Cone distance	R	$= 50m = 50 \times 3 = 150\,mm$
Mean pitch line velocity	v_m	$= 4.618\,m/sec$
Velocity factor	C_v	$= 0.7209$
Service factor	C_s	$= 1.25$

(iv) Checking

(a) Dynamic load

According to Buckingham's equation,

$$\text{Dynamic load } F_d = F_t + \frac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}} \qquad \text{----}2.440\,a$$

From Fig. 2.30 (Old DDHB) ; Fig. 23.35a (New DDHB) for $v_m = 4.168\,m/sec$

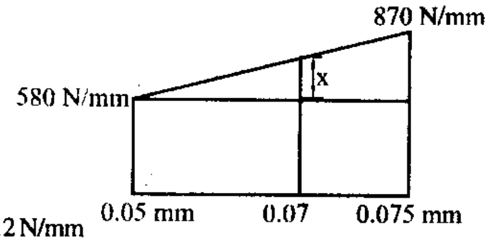
$$\text{Error } f = 0.07\,mm$$

From Table 2.35 (Old DDHB) ; Table 23.32 (New DDHB) for 20° full depth, steel-steel combination

$$\text{For error } f = 0.075\,mm;\ C = 870\,kN/m = 870\,N/mm$$

$$\text{For error } f = 0.05\,mm;\ C = 580\,kN/m = 580\,N/mm$$

By interpolation



$$\frac{x}{0.07 - 0.05} = \frac{870 - 580}{0.075 - 0.05}$$

$$x = 232\,N/mm$$

$$\therefore \text{For error } f = 0.063056\,mm;$$

$$C = 580 + 232 = 812\,N/mm$$

$$\therefore F_d = 3248.3 + \frac{21 \times 4.618[3248.3 + 30 \times 812]}{21 \times 4.618 + \sqrt{3248.3 + 30 \times 812}} = 13421.436\,N$$

b) Endurance strength

Endurance strength $F_{-1} = F_f = \sigma_{-1} by\pi m \left(\dfrac{R-b}{R}\right) = \sigma_f by\,\pi m \left(\dfrac{R-b}{R}\right)$

Endurance strength of the weaker member pinion $F_f = \sigma_f\, by_1 \pi m \left(\dfrac{R-b}{R}\right)$

From Table 23.33 (New DDHB) ; Table 2.39 (Old DDHB) for Surface hardness = 320 BHN

$$\text{Endurance limit } \sigma_{sf} = \sigma_f = \sigma_{-1} = 551.8\,N/mm^2$$

$$\therefore F_{-1} = F_f = (551.8)(30)(0.1094)(\pi \times 3)\left(\frac{150 - 30}{150}\right) = 13654.67\,N$$

As $F_{-1} = F_f > F_d$, the design is satisfactory from the stand point of strength.

c) Wear strength

According to Buckingham's equation neglecting the deflection effect

$$F_w = \frac{d_1 bQK}{\cos\delta_1}$$

$$\text{Ratio factor } Q = \frac{2Z_{v2}}{Z_{v1}+Z_{v2}} = \frac{2\times 490}{30.442+490} = 1.883$$

For carbon steel from Table 2.8 (Old DDHB – Vol- I) or Table 2.10 (New DDHB Vol-I)

$$\text{Young's modulus } E_1 = E_2 = 206\,\text{GPa} = 206\times 10^3\,\text{N/mm}^2$$

$\therefore$ Equivalent Young's modulus $E_o = 206\times 10^3\,\text{N/mm}^2$

$\therefore$ Limiting surface fatigue stress $\sigma_{fac} = \sigma_{-1c} = (2.75\,H_B - 69)$ MPa—2.291c (Old); 23.168a (New DDHB)

$$= (2.75\times 320 - 69) = 811\,\text{N/mm}^2$$

$$\text{Load stress factor } K = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_o} = \frac{1.43\times 811^2\times \sin 20}{206\times 10^3} = 1.5616\,\text{N/mm}^2$$

$$\therefore \text{ Wear load } F_w = \frac{(60)(30)(1.883)(1.5616)}{\cos(11.535)} = 5402\,\text{N}$$

As $F_w < F_d$, the design will be unsatisfactory from the standpoint of durability or wear.

For safer design, $F_w \geq F_d$

i.e., $\frac{d_1 bQK}{\cos\delta_1} \geq 13421.436$; i.e., $\frac{(60)(30)(1.883)(K)}{\cos(11.535)} \geq 13421.436$ $\therefore$ $K \geq 3.88$ N/mm²

i.e., $\frac{1.43\sigma_{fac}^2 \sin 20}{206\times 10^3} \geq 3.88$; i.e., $\sigma_{fac} \geq 1278.37\,\text{N/mm}^2$; i.e., $2.75\,H_B - 69 \geq 1278.37$; $H_B \geq 489.9$

$\therefore$ For safer design the average surface hardness of the gear pair should not be less than 500 BHN.

$\therefore$ Surface hardness for pinion = 500 BHN; Surface hardness for gear = 500 BHN

Example 5.5

A pair of straight bevel gears transmit 15 kW at 1250 rpm of 120 mm diameter pinion. The speed reduction is 3.5. Use 14½° full depth tooth system. The pinion is made of alloy steel of case hardened and (SAE 2320) gear is cast steel 0.20% C heat treated. Determine module, face width and number of teeth on pinion and gear. Suggest suitable hardness if the wear strength has to be more than the dynamic load.

Data :

$P = N = 15$ kW; $n_1 = 1250$ rpm; $d_1 = 120$ mm; $i = 3.5$; $\alpha = 14½°$

Pinion material – Alloy steel case hardened (SAE 2320)

Gear material – Cast steel 0.20% C heat treated

Solution :

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$\therefore$ Speed of gear $n_2 = \frac{n_1}{i} = \frac{1250}{3.5} = 357.14$ rpm

Pitch circle diameter of gear $d_2 = id_1 = 3.5 \times 120 = 420$ mm

Assume right angle bevel gear i.e., $\Sigma = 90°$

$\therefore$ Pitch cone angle of pinion $\delta_1 = \tan^{-1}\left(\frac{1}{i}\right) = \tan^{-1}\left(\frac{1}{3.5}\right) = 15.945°$ ---- 2.402

Pitch cone angle of gear $\delta_2 = \tan^{-1}(i) = \tan^{-1}(3.5) = 74.055°$ ---- 2.403

From ; Table 23.18 (New DDHB)

Allowable static stress of pinion $\sigma_{o1} = 345$ N/mm²

Allowable static stress of gear $\sigma_{o2} = 173$ N/mm²

To identify the weaker member temporarily assume $z_1 = 20$. $\therefore$ $z_2 = iz_1 = 3.5 \times 20 = 70$

Formative number of teeth on pinion $z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{20}{\cos 15.945}$ 20.8

Formative number of teeth on gear $z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{70}{\cos 74.055} = 254.79$

Lewis form factor for 14½° involute $y = 0.124 - \frac{0.684}{z_v}$ ---- 2.97 (Old DDHB) ; **23.115 (New DDHB)**

Form factor of pinion $y_1 = 0.124 - \frac{0.684}{z_{v_1}} = 0.124 - \frac{0.684}{20.8} = 0.0911$

Form factor of gear $y_2 = 0.124 - \frac{0.684}{z_{v_2}} = 0.124 - \frac{0.684}{254.79} = 0.1213$

(i) Identify the weaker member

Particulars	σ_o N/mm²	y	$\sigma_o y$	Remarks
Pinion	345	0.0911	31.4295	
Gear	173	0.1213	20.9849	Weaker

Since $\sigma_{02} y_2 < \sigma_{01} y_1$ gear is weaker. Therefore design should be based on gear

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000\,\text{NCs}}{nr} = \frac{9550 \times 1000\,\text{PCs}}{nr}$ where r in mm

$\therefore$ Tangential tooth load of weaker member gear $F_{t2} = \frac{9550 \times 1000\,\text{NCs}}{n_2 r_2} = \frac{9550 \times 1000\,\text{PCs}}{n_2 r_2}$

Assume moderate medium shock and 8-10 hrs duty per day

$\therefore$ From Table 2.33 (Old DDHB) ; **Table 23.13 (New DDHB)** or Table 2.86, service factor $C_S = 1.5$

Pitch circle radius of gear $r_2 = \frac{d_2}{2} = \frac{420}{2} = 210\,mm$

$$\therefore\ F_{t_2} = \frac{9550 \times 1000 \times 15 \times 1.5}{357.14 \times 210} = 2865\,N$$

b) Tangential tooth load from Lewis equation $F_t = \sigma_o C_v b Y m\left(\frac{R-b}{R}\right)$ ----2.426a (DDHB)

$$\text{For weaker member } F_{t2} = \sigma_{o2} C_v b\, \pi y_2 m\left(\frac{R-b}{R}\right)$$

Mean pitch line velocity of weaker member $v_m = \frac{\pi d_2 n_2}{60000} = \frac{\pi \times 420 \times 357.14}{60000} = 7.854$ m/sec

Assume the teeth are generated

$$\therefore\ \text{Velocity factor } C_v = \frac{5.55}{5.55 + \sqrt{v_m}} = \frac{5.55}{5.55 + \sqrt{7.854}} = 0.6645$$

$$\text{Cone distance } R = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{120^2 + 420^2} = 218.4\,mm$$

$$\text{For face width, } \frac{R}{4} < b < \frac{R}{3}$$

$$\therefore\ b = \frac{R}{3} = \frac{218.4}{3} = 72.8\,mm$$

$$\text{Take face width } b = 70mm$$

$$\text{Lewis form factor } y_2 = 0.124 - \frac{0.684}{z_{2v}} = 0.124 - \frac{0.684\cos\delta_2}{z_2}$$

$$= 0.124 - \frac{0.684\cos\delta_2 \times m}{d_2} = 0.124 - \frac{0.684\cos 74.055 \times m}{420}$$

$$= 0.124 - 4.474 \times 10^{-4} m$$

$$\therefore\ 2865 = (173)(0.6645)(70)(\pi)(0.124 - 4.474 \times 10^{-4} m)\, m\left(\frac{218.4 - 70}{218.4}\right)$$

$$\text{i.e., } 0.1668 = 0.124\,m - 4.474 \times 10^{-4} m^2$$

i.e., $4.474 \times 10^{-4} m^2 - 0.124\,m + 0.1688 = 0$

i.e., $m^2 - 277.2\,m + 372.82 = 0$

$$\therefore\ m = \frac{+277.2 \pm \sqrt{277.2^2 - 4 \times 1 \times 372.82}}{2 \times 1}$$

$$= 275.85\,mm \text{ or } 1.35\,mm$$

Select the smaller value $\therefore$ m = 1.35

From Table 2.3 (Old DDHB) ; Table 23.3 (New DDHB) select the standard module

$\therefore$ Module m = 1.5 mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{o2}C_v)_{all} = 173 \times 0.6645 = 114.96\ N/mm^2$

Induced stress $\sigma_{ind} = (\sigma_{o2}C_v)_{ind} = \dfrac{F_{t2}}{b\pi y_2 m\left(\dfrac{R-b}{R}\right)}$ ----2.426a

$$= \frac{2865}{(70)(\pi)(0.124 - 4.474 \times 10^{-4} \times 1.5)(1.5)\left(\dfrac{218.4-70}{218.4}\right)}$$

$$= 103.64\ N/mm^2$$

Since $(\sigma_{o2}C_v)_{ind} < (\sigma_{o2}C_v)_{all}$, the design is satisfactory.

Also in order to avoid the breakage of gear teeth due to bending, the beam strength should be more than the effective force between the meshing teeth.

i.e., $F_b \geq F_{eff}$

$$F_b = mb\sigma_b \pi y_2\left(1-\frac{b}{R}\right)$$

$$= (1.5)(70)(173)(\pi)(0.124 - 4.474 \times 10^{-4} \times 1.5)\left(1-\frac{70}{218.4}\right)$$

$$= 4782.24\ N$$

$F_{eff} = \dfrac{F_t C_S}{C_v} = \dfrac{F_t}{C_v}$ since C_s is already considered

$$= \frac{2865}{0.6645} = 4311.5\ N$$

Since $F_b > F_{eff}$, the design is satisfactory from the stand point of beam strength.

$\therefore$ Module m = 1.5mm

Face width b = 70mm

Cone distance R = 218.4mm

Number of teeth on pinion $z_1 = \dfrac{d_1}{m} = \dfrac{120}{1.5} = 80$

Number of teeth on gear $z_2 = \dfrac{d_2}{m} = \dfrac{420}{1.5} = 280$

Checking

According to Buckingham's equation

Dynamic load $F_d = F_t + \dfrac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}}$

From Fig. 2.30 (Old DDHB) ; Fig. 23.35a (New DDHB), for $v_m = 7.854$ m/sec

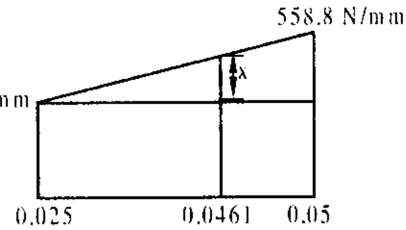
Error f = 0.0461 mm

From Table 2.35 (Old DDHB) ; Table 23.32 (New DDHB) for $\alpha = 14½°$, steel – steel combination

For error f = 0.025 mm; C = 279.48 kN/m = 279.48 N/mm

For error f = 0.05 mm; C = 558.8 kN/m = 558.8 N/mm

By interpolation



$$\frac{x}{558.8 - 279.48} = \frac{0.0461 - 0.025}{0.05 - 0.025}$$

$$\therefore x = 235.746 \text{ N/mm}$$

$\therefore$ For error f = 0.0461 mm; C = 279.48 + 235.746 = 515.226 N

$$\therefore F_d = 2865 + \frac{21 \times 7.854[2865 + 70 \times 515.226]}{21 \times 7.854 + \sqrt{2865 + 70 \times 515.226}} = 20590.7 \text{ N}$$

According to Buckingham's equation neglecting the effect of deflection

$$\text{Wear load } F_w = \frac{d_1 bQK}{\cos\delta_1} \qquad \text{----2.441 a}$$

$$\text{Ratio factor } Q = \frac{2z_{v2}}{z_{v1} + z_{v2}}; z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{96}{\cos 15.945} = 99.84$$

$$z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{336}{\cos 74.055} = 1223.09$$

$$\therefore Q = \frac{2 \times 1223.09}{99.84 + 1223.09} = 1.849$$

$$\text{Given } F_w > F_d$$

$$\text{i.e., } \frac{d_1 bQK}{\cos\delta_1} > F_d$$

$$\text{i.e., } \frac{120 \times 70 \times 1.849 \times K}{\cos 15.945} > 20590.7$$

$$\therefore \text{Load stress factor } K > 1.2747 \text{ N/mm}^2$$

From Table 2.40 (Old DDHB) ; **Table 23.37B (New DDHB)** for $\alpha = 14½°$ and load stress factor K = 1.2747 N/mm^2

Hardness of pinion = 400 BHN

Hardness of gear = 300 BHN

Example 5.6

A pair of bevel gear wheels with 20° pressure angle consist of 20 teeth pinion meshing with 30 teeth gear. The module is 4mm while the face width is 20mm. The surface hardness of both pinion and gear is 400 BHN. The pinion rotates at 500rpm and receives power from an electric motor. The starting torque of the motor is 150 percent of the rated torque. Determine the safe power that can be transmitted considering the dynamic load, wear strength and endurance strength. The allowable bending stress may be taken as 240 MPa.

(VTU Feb. 2002)

Data :

$\alpha = 20°$ (Assume full depth); $z_1 = 20$; $z_2 = 30$; m = 4mm; b = 20 mm

Surface hardness of pinion and gear = 400 BHN; $n_1 = 500$rpm; $C_s = 1.5$; $\sigma_{b1} = \sigma_{b2} = 240$MPa

Solution :

$$\text{Velocity ratio} = i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore i = \frac{z_2}{z_1} = \frac{30}{20} = 1.5$$

$$\therefore \text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{500}{1.5} = 333.33 \text{ rpm}$$

Assume right angle bevel gear. $\therefore$ shaft angle $\Sigma = 90°$

For right angle bevel gear

$$\text{Pitch cone angle of pinion } \delta_1 = \tan^{-1}\left(\frac{1}{i}\right) = 33.69° \qquad \text{---- 2.402 (DDHB)}$$

$$\text{Pitch cone angle of gear } \delta_2 = \tan^{-1}(i) = \tan^{-1}(1.5) = 56.31° \qquad \text{---- 2.403 (DDHB)}$$

$$\text{Formative number of teeth on Pinion } z_{v1} = \frac{z_1}{\cos\delta_1}$$

$$= \frac{20}{\cos 33.69} = 24.04$$

$$\text{Formative number of teeth on gear } z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{30}{\cos 56.31} = 54.08$$

$$\text{Lewis form factor for } 20° \text{ full depth, } y = 0.154 - \frac{0.912}{z_v} \qquad \text{---- 2.98 (Old DDHB) ; 23.116 (New DDHB)}$$

$$\therefore \text{Form factor for pinion } y_1 = 0.154 - \frac{0.912}{z_{v1}}$$

$$= 0.154 - \frac{0.912}{24.04} = 0.11606$$

$$\text{Form factor for gear } y_2 = 0.154 - \frac{0.912}{z_{v2}}$$

$$= 0.154 - \frac{0.912}{54.08} = 0.13714$$

As the same material is used for both pinion and gear, the pinion is weaker than the gear. Therefore design should be based on pinion.

Pitch circle diameter of pinion $d_1 = mz_1 = 4 \times 20 = 80mm$

Pitch circle diameter of gear $d_2 = mz_2 = 4 \times 30 = 120mm$

$$\text{Cone distance } R = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{80^2 + 120^2} = 72.11mm \qquad \text{----2.414}$$

Beam Strength

According to Lewis equation

$$\text{Beam strength } F_b = mb\sigma_b Y\left(1 - \frac{b}{R}\right)$$

$$\therefore \text{ Beam strength of weaker member } F_{b1} = mb\sigma_{b1}(\pi y_1)\left(1 - \frac{b}{R}\right)$$

$$= 4 \times 20 \times 240(\pi \times 0.11606)\left(1 - \frac{20}{72.11}\right) = 5058.96\,N$$

Wear Strength

According to Buckingham's equation, Wear strength $F_w = \dfrac{d_1 bQK}{\cos\delta_1}$ ----2.441 a (DDHB)

In the case of bevel gears, either the pinion or the gear is generally overhanging and hence it is subjected to deflection under the action of tooth forces. Considering this effect wear strength

$$F_w = \frac{0.75 d_1 bQK}{\cos\delta_1}$$

$$\text{Ratio factor } Q = \frac{2z_{v2}}{z_{v1} + z_{v2}} = \frac{2 \times 54.08}{24.04 \times 54.08} = 1.38454$$

Limiting stress for surface fatigue $\sigma_{fac} = \sigma_{-1c} = (2.75\,H_B - 69)$ MPa ---- 2.291c (Old) ; 23.168a (New DDHB)

$= (2.75 \times 400 - 69) = 1031\ N/mm^2$

Assume the material for both pinion and gear are steel

Therefore from Table 2.8 (Old DDHB-Vol. I) : Table 2.10 (New DDHB - Vol. I), $E_1 = E_2 = 206 \times 10^3\ N/mm^2$

$$\text{Equivalent Young's modulus } E_0 = \frac{2E_1E_2}{E_1 + E_2} = 206 \times 10^3\ N/mm^2$$

$$\therefore \text{ Load stress factor } K = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_0} \qquad \text{---- 23.161 (New DDHB)}$$

$$= \frac{1.43 \times 1031^2 \times \sin 20}{206 \times 10^3} = 2.5237\ N/mm^2$$

$$\text{Effective wear strength } F_w = \frac{(0.75)(80)(20)(1.38454)(2.5237)}{\cos 33.69} = 5039.35\,N$$

Dynamic load

According to Prof M.H. Spotts

$$\text{Dynamic load } F_d = \frac{e n_1 z_1 b' r_1 r_2}{2530\sqrt{r_1^2 + r_2^2}} \text{ for steel pinion and gear}$$

where e = Total error 'f'

n_1 = Speed of pinion = 500rpm

z_1 = Number of teeth on pinion = 24

b' = Axial width of gear blank $b\cos\delta_1 = 20\cos 33.69 = 16.641$mm

r_1 = Pitch circle radius of pinion $= \frac{d_1}{2} = \frac{80}{2} = 40$ mm

r_1 = Pitch circle radius of gear $= \frac{d_2}{2} = \frac{120}{2} = 60$mm

Mean pitch line velocity of the weaker member pinion

$$v_m = \frac{\pi d_1 n_1}{60{,}000} = \frac{\pi \times 60 \times 500}{60{,}000} = 1.5708 \text{ m/sec}$$

Assume the gears are First class commercial gears

Therefore from Fig 2.29 (Old DDHB); Fig. 23.34a (New DDHB) for module m = 4mm, error f = 0.05mm

$$\therefore \text{ Dynamic load } F_d = \frac{(0.05)(500)(20)(16.641)(40)(60)}{2530\sqrt{40^2 + 60^2}} = 109.456\,N$$

Endurance strength

$$\text{Endurance strength of the weaker member } F_f = F_{-1} = \sigma_{-1}\, b y_1 \pi m \left(\frac{R-b}{R}\right) = \sigma_{sf}\, b y_1 \pi m \left(\frac{R-b}{R}\right)$$

From Table 23.33, for BHN = 400, $\sigma_{sf} = \sigma_f = 689.6$ N/mm^2

$$\therefore \text{ Endurance strength } F_{-1} = (689.6)(20)(0.11606)(\pi \times 4)\left(\frac{72.11 - 20}{72.11}\right) = 14536\,N$$

Safe power

In order to avoid failure of the gear tooth due to bending $F_b = F_{eff}(FOS)$ where $F_{eff} = C_s F_t + F_d$ considering the factor of safety as unity $F_b = C_s F_t + F_d$

In order to avoid failure of the gear tooth due to pitting $F_w = F_{eff}$ (FOS) considering FOS as unity

$$F_w = C_s F_t + F_d$$

In order to avoid failure of the gear tooth due to fatique $F_f = F_{-1} = F_{eff}$ (FOS). Considering FOS as unity $F_{-1} = C_s F_t + F_d$.

As wear strength is lower than beam strength and endurance strength, wear failure (due to pithing) is the criterion of design.

$$\therefore F_w = C_s F_t + F_d$$

$$\text{i.e., } 5039.35 = 1.5 F_t + 109.456$$

$\therefore$ Tangential tooth load of the weaker member $F_t = 3286.6$ N

$$\text{Also } F_{t1} = \frac{9550 \times 1000 \times N}{n_1 r_1} = \frac{9550 \times 1000 \times P}{n_1 r_1}$$

$$\text{i.e., } 3286.6 = \frac{9550 \times 1000 \times P}{500 \times 40}$$

$\therefore$ Safe power transmitted P = 6.883 kW (If Fos = 1)

Example 5.7

Two steel bevel gears connect shafts at 90°, the pinion has a surface hardness of 300 BHN and the gear has a surface hardness of 200 BHN. The tooth profile is to be of the $14\frac{1}{2}^\circ$ full depth involute and module is 4 mm. The number of teeth on the pinion is 30 and the gear has 48 teeth. The face width is 40 mm. Determine the wear strength of the pair. What would be the power that can be transmitted based on the above wear strength if the pinion rotates at 1440 rpm **VTU, August 2004**

Data :

$$\Sigma = 90°; \text{ Surface hardness of pinion} = 300 \text{ BHN}$$

Surface hardness of gear = 200 BHN; $\alpha = 14\frac{1}{2}^\circ$; m = 4 mm; $z_1 = 30$; $z_2 = 48$; b = 40 mm; $n_1 = 1440$ rpm

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore i = \frac{z_2}{z_1} = \frac{48}{30} = 1.6$$

$$\text{Speed of pinion } n_2 = \frac{n_1}{i} = \frac{1440}{1.6} = 900 \text{ rpm}$$

For right angle bevel gear, pitch cone angle of pinion $\delta_1 = \tan^{-1}\left(\frac{1}{i}\right) = \tan^{-1}\left(\frac{1}{1.6}\right) = 32°$

$$\text{Pitch cone angle of gear } \delta_2 = \tan^{-1}(i) = \tan^{-1} 1.6 = 58°$$

$$\text{Pitch circle diameter of pinion } d_1 = m z_1 = 4 \times 30 = 120 \text{ mm}$$

$$\text{Formative number of teeth on pinion } z_{v_1} = \frac{z_1}{\cos\delta_1} = \frac{30}{\cos 32} = 35.3754$$

$$\text{Formative number of teeth on gear } z_{v_2} = \frac{z_2}{\cos\delta_2} = \frac{48}{\cos 58} = 90.58$$

$$\therefore \text{Ratio factor } Q = \frac{2 z_{v_2}}{2 z_{v_1} + z_{v_2}} = \frac{2 \times 90.58}{35.3754 + 90.58} = 1.4383$$

$$\text{Average value of BHN for the gear pair} = \frac{300+200}{2} = 250$$

Limiting stress for surface fatigue $\sigma_{fac} = \sigma_{-1C} =$ $(2.75\,H_B - 69)\,MPa = 2.75 \times 250 - 69 = 618.5\ N/mm^2$

Assume same material (steel) is used for both pinion and gear. Therefore pinion is weaker than the gear and hence design should be based on pinion.

$$\text{For steel } E_1 = E_2 = 206 \times 10^3\ N/mm^2$$

$$\therefore E_o = \frac{2E_1E_2}{E_1+E_2} = 206 \times 10^3\ N/mm^2$$

$$\therefore \text{Load stress factor } K = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_o} = \frac{1.43 \times 618.5^2 \times \sin 14\frac{1}{2}}{206 \times 10^3}$$

$$= 0.665\ N/mm^2$$

According to Buckingham's equation neglecting overhanging and deflection effect,

$$\text{Wear load } F_w = \frac{d_1 bQK}{\cos\delta_1} \qquad \text{----2.441 a (DDHB)}$$

$$= \frac{(120)(40)(1.4383)(0.665)}{\cos 32} = 5413.67\ N$$

$\therefore$ Neglecting overhanging and deflection effect, wear load $F_w = 5413.67$ N

In order to avoid failure of the gear tooth due to pitting

$$F_w = (F_{eff})(FOS) = (C_sF_t + F_d)\,FOS$$

According to Prof MH Spotts

$$\text{Dynamic load } F_d = \frac{en_1z_1b'\,r_1r_2}{2530\sqrt{r_1^2 + r_2^2}} \text{ for steel pinion and gear}$$

where e = Total error 'f'

b' = Axial width of gear blank $= b\cos\delta_1 = 40 \cos 32 = 33.922$ mm

r_1 = Pitch circle radius of pinion $= \frac{d_1}{2} = \frac{120}{2} = 60$ mm

r_2 = Pitch circle radius of gear $= \frac{d_2}{2} = \frac{mz_2}{2} = \frac{4 \times 48}{2} = 96$ mm

Assume the gears are first class commercial gears

$\therefore$ From Fig. 2.29 (Old DDHB) ; Fig. 23.34a (New DDHB) for module m = 4 mm; error f = 0.05 mm = e

$$\therefore F_d = \frac{(0.05)(1440)(30)(33.922)(60)(96)}{2530\sqrt{60^2 + 96^2}} = 1473.54\ N$$

Assume service factor C_s = 1.5 and FOS as unity

$$\therefore F_w = C_sF_t + F_d$$

$$\text{i.e., } 5413.67 = 1.5F_t + 1473.54$$

$\therefore$ Tangential load of the weaker member $F_t = 2626.75$ N

$$\text{Also } F_{t_1} = \frac{9550 \times 1000 \times N}{n_1 r_1} = \frac{9550 \times 1000 \times P}{n_1 r_1}$$

$$\text{i.e., } 2626.75 = \frac{9550 \times 1000 \times P}{1440 \times 60}$$

$\therefore$ Power transmitted P = N = 23.76 kW

$\therefore$ If factor of safety is unity then the power transmitted based on wear strength P = N = 23.76 kW

Example 5.8

A pair of mitre gears have pitch diameter 280 mm and face width of 36 mm and run at 250 rpm. The teeth are of 14½° involute and accurately cut and transmit 6 kW. Neglecting friction angle find the following

i) Outside diameter of gears

ii) Resultant tooth load tangent to pitch cone

iii) Radial load on the pinion

iv) Thrust on the pinion

VTU, July/Aug 2002

Data:

Mitre gear; d = 280 mm; b = 36 mm; n = 250 rpm

α = 14½ ; Precision cut; P = N = 6 kW

Solution :

For mitre bevel gear

$$d = d_1 = d_2 = 280 \text{ mm}$$
$$n = n_1 = n_2 = 250 \text{ rpm}$$
$$z_1 = z_2$$
$$\delta_1 = \delta_2 = 45°$$
$$\text{Shaft angle } \Sigma = 90°$$
$$\text{Velocity ratio } i = 1$$

(i) Outside diameter of gears

Assume the material for both the gears are low carbon cast steel 0.2% C heat treated. From **Table 23.18 (New DDHB)**

Allowable static stress $\sigma_{01} = \sigma_{02} = 173$ MPa = 173 N/mm²

Lewis form factor for 14½° involute $y = 0.124 - \frac{0.684}{z_v}$

Lewis form factor for pinion $y_1 = 0.124 - \frac{0.684}{z_{v1}} = 0.124 - \frac{0.684 \times \cos\delta_1}{z_1}$

$$= 0.124 - \frac{0.684 \times \cos\delta_1 \times m}{d_1} = 0.124 - \frac{0.684 \times \cos 45 \times m}{280}$$

$$= 0.124 - 1.72736 \times 10^{-3} m = y_2$$

$$\text{Cone distance } R = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{280^2 + 280^2} = 198 \text{ mm} \qquad \text{----2.414 (DDHB)}$$

$$\text{Mean pitch line velocity} \quad v_m = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 280 \times 250}{60000} = 3.6652 \text{ m/sec}$$

Assume the teeth are generated

$$\therefore \text{ Velocity factor } C_v = \frac{5.55}{5.55 + \sqrt{v_m}} = \frac{5.55}{5.55 + \sqrt{3.6652}} = 0.74352$$

$$\text{Tangential tooth load } F_t = \frac{9550 \times 1000 P C_s}{nr} \text{ where r in mm and assume service factor } C_s = 1.5$$

$$\therefore \text{ Tangential tooth load on the pinion } F_{t1} = \frac{9550 \times 1000 \times 6 \times 1.5}{250 \times \left(\frac{280}{2}\right)} = 2455.7143 \text{ N}$$

$$\text{Tangential tooth load from Lewis equation } F_t = \sigma_o b Y C_v m \left(\frac{R-b}{R}\right)$$

$$\text{For pinion } F_{t1} = \sigma_o b \pi y_1 C_v m \left(\frac{R-b}{R}\right) \qquad \text{----2.426 a (DDHB)}$$

$$\text{i.e., } 2455.7143 = (173)(36)(\pi)(0.124 - 1.72736 \times 10^{-3} m)(0.74352)(m)\left(\frac{198-36}{198}\right)$$

$$\text{i.e., } 0.20632 = 0.124 m - 1.72736 \times 10^{-3} m^2$$

$$\text{i.e., } m^2 - 71.8 m + 119.442 = 0$$

$$\therefore m = \frac{+71.8 \pm \sqrt{71.8^2 - 4 \times 1 \times 119.442}}{2 \times 1} = 70.1 \text{ mm or } 1.7 \text{ mm}$$

Selecting the smaller value, module m = 1.7 mm

From Table 2.3 (Old DDHB) ; Table 23.3 (New DDHB) select the standard module

$\therefore$ **Module m = 2 mm**

Check for the stress

$$\text{Allowable stress } \sigma_{all} = (\sigma_{02} C_v)_{all} = (173)(0.74352) = 128.63 \text{ N/mm}^2$$

$$\text{Induced stress } \sigma_{ind} = (\sigma_{02} C_v)_{ind} = \frac{F_{t1}}{b \pi y_1 m \left(\frac{R-b}{R}\right)} \qquad \text{----2.426a (DDHB)}$$

$$= \frac{2455.7143}{36 \times \pi \times (0.124 - 1.72736 \times 10^{-3} \times 2)(2)\left(\frac{198-36}{198}\right)}$$

$$= 110.08 \text{ N/mm}^2$$

Since $(\sigma_{01} C_v)_{ind} < (\sigma_{01} C_v)_{all}$, the design is satisfactory.

Also in order to avoid the breakage of gear teeth due to bending, the beam strength should be more than the effective force between the meshing teeth i.e., $F_b \geq F_{eff}$.

Beam strength $F_b = mb\sigma_b \pi y_1 \left(1 - \frac{b}{R}\right)$

$= (2)(36)(173)(\pi)(0.124 - 1.72736 \times 10^{-3} \times 2)\left(1 - \frac{36}{198}\right)$

$= 3859.48\,N$

Effective load or force $F_{eff} = \frac{F_t C_s}{C_v} = \frac{F_t}{C_v}$ since C_s is already considered

$= \frac{2455.7143}{0.74352} = 3302.82\,N$

As $F_b > F_{eff}$, the design is satisfactory from the stand point of beam strength.

$\therefore$ Module m = 2mm

Face width b = 36mm

Cone distance R = 198mm

Number of teeth on pinion $z_1 = \frac{d_1}{m} = \frac{280}{2} = 140$

Number of teeth on gear $z_2 = 140$

From Table 2.1 (Old DDHB); Table 23.1 (New DDHB)

Addendum $h_a = 1m = 1 \times 2 = 2\,mm$

$\therefore$ Outside diameter of pinion $da_1 = d_1 + 2\,ha = 280 + 2 \times 2 = 284\,mm$

Outside diameter of gear $da_2 = 284\,mm$

ii) Resultant tooth load tangent to pitch cone

Mean radius of pinion $r_m = \left[\frac{d_1 - b\sin\delta_1}{2}\right] = \left[\frac{280 - 36\sin 45}{2}\right] = 127.272\,mm$

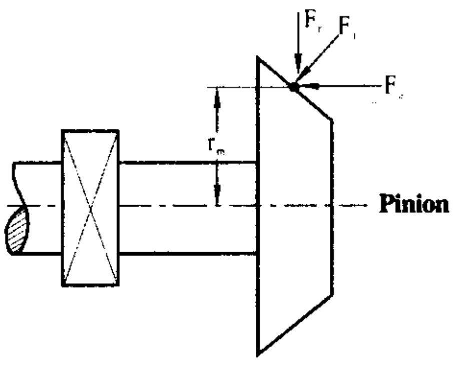


Fig. 5.11

Torque on the pinion shaft $M_{t1} = \frac{60 \times 10^6 \times PC_S}{2\pi n_1}$ [Considering the service factor]

$$= \frac{60 \times 10^6 \times 6 \times 1.5}{2\pi \times 250} = 343774.68 \text{ Nmm}$$

$\therefore$ Tangential force at the mean radius $F_{t_m} = \frac{M_{t1}}{r_m} = \frac{343774.68}{127.272} = 2701.1 \text{ N}$

iii) Radial load on the pinion

$$F_r = F_{t_m} \tan\alpha \cos\delta_1 = 2701.1 \times \tan 14½ \times \cos 45 = 493.95 \text{ N}$$

iv) Thrust load on the pinion

$$F_a = F_{t_m} \tan\alpha \sin\delta_1 = 2701.1 \times \tan 14½ \times \sin 45 = 493.95 \text{ N}$$

NOTE : According to Prof. M.F. Spotts

In order to avoid failure of gear tooth due to bending in the initial stages of design

$$F_b > F_{eff}$$

Introducing factor of safety $F_b = F_{eff}(FOS)$ ----(i)

where F_b = Beam strength $= mb\sigma_b Y\left(1 - \frac{b}{R}\right)$

F_{eff} = Effective load $= \frac{F_t C_s}{C_v}$ ----(ii)

σ_b = Allowable bending stress $\left(= \frac{\sigma_u}{3}\right)$ of the weaker member

b = Face width

R = Cone distance

m = module

C_v = Velocity factor

C_s = Service factor

σ_u = Ultimate strength; $Y = \pi y$

FOS = factor of safety; y = Lewis form factor of the weaker member

$$F_t = \frac{9550 \times 1000 N}{nr}$$

r = Pitch circle radius of weaker member in mm

n = Speed of the weaker member in rpm

The module obtained from equation (i) is based on beam strength

Considering dynamic load according to spotts

$F_{eff} = C_s F_t + F_d$ ---(iii)

where F_d = Dynamic load $= \frac{e n_1 z_1 b' r_1 r_2}{2530\sqrt{r_1^2 + r_2^2}}$ for steel pinion and steel gear

e = total error (f)

b' = axial width of gear blank $= b\cos\delta_1$

Considering the dynamic load to avoid failure of the gear tooth due to bending

$$F_b = F_{eff}(FOS)$$

$$= (C_s F_t + F_d)(FOS) \quad \text{---- (iv)}$$

Wear load $F_w = \frac{d_1 bQK}{\cos\delta_1}$

Considering the effect of overhanging and deflection, effective wear load $F_w = \frac{0.75 d_1 bQK}{\cos\delta_1}$

To avoid failure of gear tooth due to pitting

$$F_w = F_{eff}(FOS)$$

$$= (C_s F_t + F_d)(FOS) \quad \text{---- (v)}$$

$$K = \frac{1.43\sigma_{-1c}^2 \sin\alpha}{E_o}; \; Q = \frac{2z_{v2}}{z_{v1} + z_{v2}}; \; E_o = \frac{2E_1E_2}{E_1 + E_2}$$

The gear can also be designed using Prof. MF Spotts equations.

Also according to American Gear Manufactures Association (AGMA) for straight tooth bevel gears.

i) Peak load power rating $N = \frac{m\sigma_o n_1 d_1 by\pi}{19100 \times 1000}\left(\frac{R - 0.5b}{R}\right)\left[\frac{5.55}{5.55 + \sqrt{v_m}}\right]$

σ_o = Allowable stress of the weaker member in N/mm²

y = form factor of the weaker member

m = module in mm

n_1 = speed of pinion

d_1 = pitch circle diameter of pinion in mm

b = face width in mm

R = Cone distance in mm

v_m = mean pitch line velocity in m/sec.

N = power in kW

Power rating based on wear (durability) $N = 0.8\, K_M C_B b$

where $C_B = \frac{d_1^{1.5} n_1}{0.032}\left(\frac{5.55}{5.55 + \sqrt{v_m}}\right)$ where d_1 in meters.

K_m = Material factor from table 2.90

or Based on wear or durability $N = 23.5\, K_m C_B . b$ ---- 2.448 a

where $C_B = 10\sqrt{d_1 d_2}\, n'_1 \left[\frac{5.55}{5.55 + \sqrt{v_m}}\right]$

d_1, d_2 are in meter $n'_1 = \frac{n}{60}$ (speed in rps)

v_m in m/sec.; b = face width in meters

Design horse power $N_d = NC_s$ ----2.451

where C_s = Service factor from table 2.86

K_m = Material factor from table 2.90

N = Rated power in kW

5.8 WORM GEAR

Worm gears are similar to crossed helical gears. It is shown in Fig. 5.12

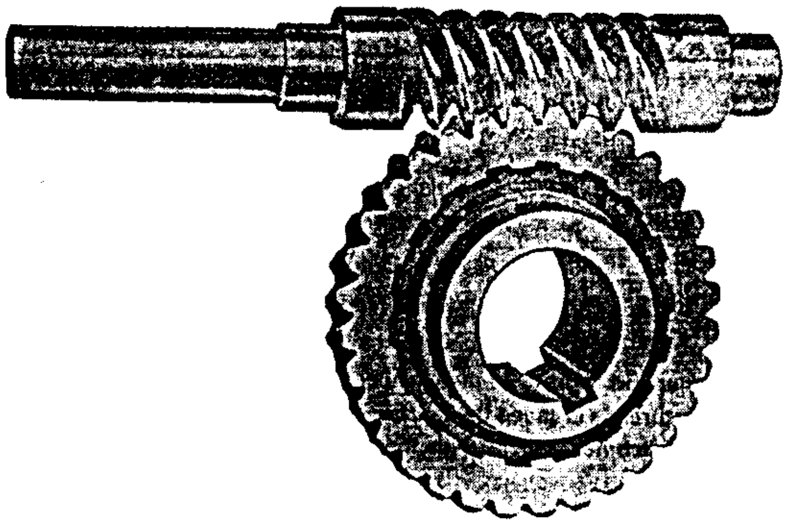
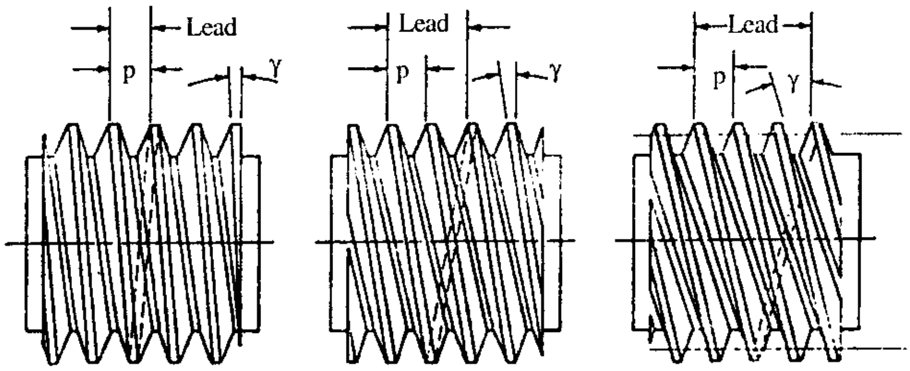


Fig. 5.12

The pinion or worm has a smaller number of teeth, usually one to four, and since they completely wrap around the pitch cylinder they are called threads. Its mating gear is called a worm gear, which is not a true helical gear.

A worm and worm gear are used to provide a high angular velocity reduction between non-intersecting shafts which are usually at right angles. The worm gear is not a helical gear because its face is made concave to fit the curvature of the worm in order to provide line contact instead of point contact. Because of the line contact, worm gearing can transmit high tooth loads. The disadvantage of worm gearing is the high sliding velocities across the teeth.



(a) *(b)* *(c)*

Fig. 5.13

Fig. 5.13 a shows a single thread worm or single start worm.

Fig. 5.13 b shows a double thread worm or double start worm.

Fig. 5.13 c shows a triple thread worm or triple start worm.

Fig 5.14 shows the nomenclature of a single enveloping worm gear set.

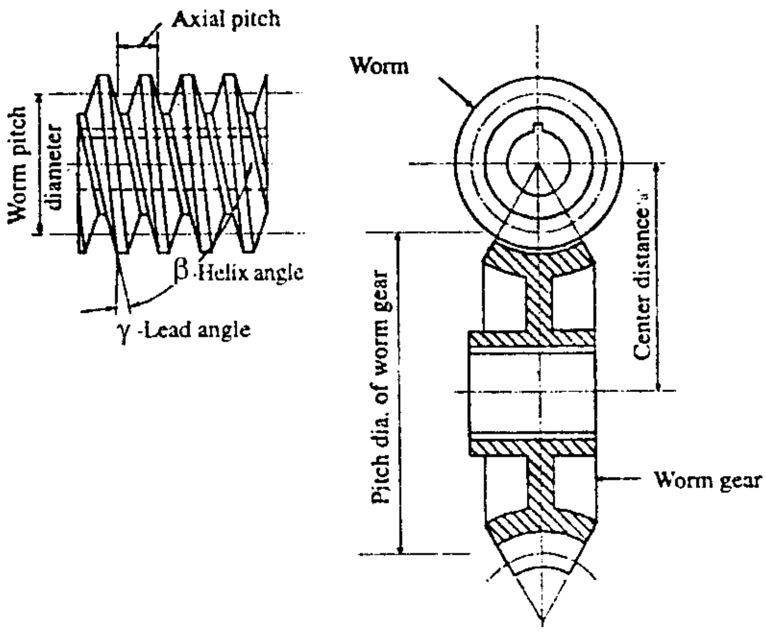


Fig. 5.14

(i) Lead : It is the amount the helix advances axially for one turn about the pitch cylinder and is equal to the number of threads times the pitch.

$\therefore$ Lead, $p_z = p_x.z_1 = \pi m_x z_1$ ---- 2.503 (DDHB)

where p_z = Lead

p_x = Axial pitch

z_1 = Number of threads (or) starts on the worm.

ii) Lead angle : The slope of the thread is called the lead angle, 'γ'

$\therefore$ $\tan\gamma = \frac{p_z}{\pi d_1}$; where d_1 = Pitch diameter of worm.

iii) Centre distance $a = \frac{d_1 + d_2}{2}$, where d_2 = Pitch diameter of worm gear.

iv) Circular pitch : The circular pitch of the worm gear must be the same as the axial pitch of the worm and can be expressed in the same manner as for a spur gear.

$\therefore$ Circular pitch $p = \frac{\pi d_2}{z_2}$ where z_2 = Number of teeth on worm gear.

v) Velocity ratio $i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{\pi d_2}{p_z}$ ---- 2.502 (DDHB)

5.9 STRENGTH OF WORM GEAR

As always worm wheel is the weaker member, design should be based on worm wheel.

Strength of worm wheel based on Lewis equation

$F_{t2} = \sigma_{02} b m_n Y C_v$ ---- 2.539 (DDHB)

where $Y = \pi y_2$, y_2 = Lewis form factor for worm wheel.

m_n = Normal module

b = Face width $\leq 0.75\, d_1$ for $z_1 = 1$ to 3 ---- 2.538 a (DDHB)

$\leq 0.67\, d_1$ for $z_1 = 4$ ---- 2.538 b (DDHB)

σ_{02} = Allowable static stress for worm wheel - Table 2.106 (DDHB)

C_v = Velocity factor $= \frac{6}{6 + v_m}$ ---- 2.542 b (DDHB)

(Always consider dynamic effect]

F_{t2} = Tangential tooth load $= \frac{9550 \times 1000 \times NC_s}{n_2 r_2} = \frac{9550 \times 1000 \times PC_s}{n_2 r_2}$

n_2 = Speed of worm wheel or worm gear in rpm ---- 23.87b (DDHB)

r_2 = Pitch circle radius of worm wheel

v_m = Mean pitch line velocity of worm gear $= \frac{\pi d_2 n_2}{60000}$ m/sec

5.10 DYNAMIC LOAD

The dynamic load of the worm gear may be found by using the following relation

$$F_d = \left(\frac{6+v_m}{6}\right)F_t = \frac{F_t}{C_v}$$

where F_t = Actual transmitted load; C_v = Velocity factor

5.11 ENDURANCE STRENGTH

The endurance strength based on Lewis equation is $F_f\ F_{-1} = \sigma_{02} b y_2 \pi m_n$

5.12 WEAR LOAD

According to Buckingham's equation the approximate wear load $F_w = d_2 bK$ ---- 2.557 (DDHB)

d_2 = Pitch circle diameter of worm gear

b = Face width of worm gear

K = Load stress factor from Table 2.111 (DDHB)

5.13 EFFICIENCY OF WORM GEARING

The efficiency of worm gearing may be defined as the ratio of output power to the input power

Efficiency when the worm drives the worm gear

$$\eta = \frac{\tan\gamma(\cos\alpha_n\cos\gamma - \mu\sin\gamma)}{\cos\alpha_n\sin\gamma + \mu\cos\gamma} \qquad \text{---- 2.587 (DDHB)}$$

Efficiency when the worm gear drives the worm

$$\eta = \frac{\cos\alpha_n\sin\gamma - \mu\cos\gamma}{\tan\gamma[\cos\alpha_n\cos\gamma + \mu\sin\gamma} \qquad \text{---- 2.588 (DDHB)}$$

where α_n = Normal pressure angle

γ = Lead angle

μ = coefficient of friction

v_r = Rubbing velocity $= \frac{\pi d_1 n_1}{60000\cos\gamma}$ ---- 2.544 a (DDHB)

$\mu = \frac{0.0422}{V_r^{0.28}}$ for $0.2 < V_r < 2.75$ m/sec ---- 2.543 a (DDHB)

$= 0.025 + \frac{V_r}{305}$ for $2.75 < V_r < 20$ m/sec ---- 2.543 b (DDHB)

Also μ can be obtained from Table 2.107 (DDHB)

5.14 THERMAL RATING OF WORM GEARING

In worm gearing the heat generated due to the work lost in friction must be dissipated in order to avoid over heating of the drive and lubricating oil

$$\text{Heat generated } H_g = \frac{\mu F_n v_r}{1000} \text{ kW} \qquad \text{---- 2.576 a (DDHB)}$$

$$\text{where } F_n = \text{Normal force} = \frac{2M_{t2}}{d_2 \cos\gamma \cos\alpha} = \frac{F_{t2}}{\cos\gamma \cos\alpha} \qquad \text{---- 2.546 (DDHB)}$$

v_r = Rubbing velocity

The generated heat must be dissipated through the lubricating oil to the gear box housing and then to the atmosphere. The heat dissipation capacity depends upon the area of the housing, temperature difference between the housing surface and surrounding air and heat transfer coefficient of the housing material.

$$\therefore \text{ Heat dissipated } H_d = \frac{h_{cr} A(t_g - t_a)}{1000} \text{ kW} \qquad \text{---- 2.577 (DDHB)}$$

where A = Radiating area of housing in $m^2 = 14.4a^{1.7}$

a = Centre distance in meter

t_g = Temperature of gear in °C

t_a = Ambient or Room temperature in °C

h_{cr} = Coefficient of heat transfer from Fig. 2.73 (DDHB)

5.15 TERMINOLOGY OF WORM GEARS

A pair of worm gears is specified and designated by four quantities in the following manner, z_1 / z_2/ q /m where

z_1 = Number of starts of warm

z_2 = Number of teeth on the worm wheel

m = Module (axial module)

q = Diametral quotient $= \frac{d_1}{m}$

d_1 = Pitch circle diameter of worm

$= \frac{a^{0.875}}{1.5}$ where d_1 and a are in mm ---- 2.519a (DDHB)

$\approx 3p_c \approx 3\pi m$

d_2 = Pitch circle diameter of worm wheel $= mz_2$

p_x = Axial pitch $= \pi m$

p_z = Lead $= p_x . z_1$

γ = Lead angle; $\tan\gamma = \frac{p_z}{\pi d_1} = \frac{\pi m z_1}{\pi d_1} = \frac{m z_1}{d_1} = \frac{m z_1}{qm} = \frac{z_1}{q}$

$$a = \text{Centre distance} = \frac{d_1 + d_2}{2}$$

$0.9\ a^{0.875} \le d_1 \le 0.5\ a^{0.875}$ ---- 2.518 a

Approximate centre distance $a = 8\ [(i + 5)\ N]^{0.588}$

$P = N$ = power transmitted in kW

b = Face width $\le 0.75\ d_1$ for $z_1 = 1$ to 3 ---- 2.538 a

$\le 0.67\ d_1$ for $z_1 = 4$ ---- 2.538 b

Also from Table 2.103 (DDHB)

Length of worm L_1 – From Table 2.95 or Table 2.104 (DDHB)

$da_1 = m\ (q + 2)$ = Outside diameter of worm

$da_2 = m\ (z_2 + 4 \cos \gamma - 2)$ = Outside diameter of worm gear

$df_1 = m\ (q + 2 - 4.4 \cos \gamma)$ = Root diameter of worm

$df_2 = m\ (z_2 - 2 - 0.4 \cos \gamma)$ = Root diameter of worm gear

m_n = Normal module = $m \cos \gamma$

5.16 AGMA POWER RATING EQUATIONS

AGMA power rating based on wear $N = \frac{n_1}{i} KQ\ C_v$ kW

where N = Power in kW

n_1 = Speed of worm

i = Velocity ratio = $\frac{n_1}{n_2}$

$$Q = \frac{i}{i + 2.5}$$

C_v = Velocity factor based on centre distance, transmission ratio and worm speed

$$= \frac{2.3}{2.3 + v_w + \frac{3v_w}{i}}$$

v_w = Pitch line velocity of worm = $\frac{\pi d_1 n_1}{60000}$ m/sec

K = Pressure constant based on center distance

Centre distance, 'a' mm	$K\left(\frac{kW}{rpm}\right)$	Centre distance mm	$K\left(\frac{kW}{rpm}\right)$
50	0.0184	375	2.94
100	0.0661	500	5.87
125	0.125	750	21.3
150	0.213	1000	48.5
200	0.485	1500	147
250	0.881	1750	235
		2000	235

AGMA power rating based on heat dissipation $N = \frac{3650a^{1.7}}{i+5}$

where a = Centre distance in meters

i = Velocity ratio

5.17 SELF LOCKING IN WORM GEARING

The efficiency of the worm gearing may be defined as the ratio of work done by the worm gear to the work done by the worm. If the efficiency of worm gearing is less than 50% then the worm gearing is said to be self locking. [i.e., it cannot be driven by applying a torque to the wheel]. This property of self locking is desirable in some applications such as hoisting machinery.

Example 5.9

Complete the design and determine the input capacity of a worm gear speed reducer unit which consists of a hardened steel worm and a phosphor bronze gear having 20° stub involute teeth. The centre distance is to be 200 mm and transmission ratio is 10 and the worm speed is 2000 rpm

VTU, July/Aug 2003

Data :

Worm material – Hardened steel

Worm gear material – Phosphor bronze

α = **20° Stub involute; i = 10; a = 200 mm;** $n_1 = 2000$ **rpm**

Solution :

i) Dimension of worm and worm gear

Velocity ratio $i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{\pi d_2}{P_z}$

Speed of worm gear $n_2 = \frac{n_1}{i} = \frac{2000}{10} = 200$ rpm

Pitch diameter of worm $d_1 = \frac{a^{0.875}}{3.5}$ where a = Centre distance in meters ----2.519 b

$= \frac{(0.2)^{0.875}}{3.5} = 0.06987 \text{ m} = 69.87 \text{ mm}$

≈ 70 mm

Centre distance $a = \frac{d_1 + d_2}{2}$

i.e., $200 = \frac{70 + d_2}{2}$

$\therefore$ Pitch diameter of worm gear $d_2 = 330$ mm, based on the first estimate

Also $d_1 \approx 3\, p_c$

i.e., $70 \approx 3 \times \pi\, m$

$\therefore$ Axial module $m = 7.42 \text{ mm} \approx 8$ mm (select standard module from Table 2.3) (Old DDHB) (Table 23.1 New DDHB)

Now $i = \frac{\pi d_2}{p_z} = \frac{\pi d_2}{\pi m_x z_1} = \frac{d_2}{m z_1}$ ($\because$ $m_x = m$ = Axial module)

$\therefore d_2 = i m z_1 = 10 \times 8 \times z_1 = 80\, z_1$

Thus for various values of z_1, the value of d_2 is tabulated as below

Z_1	1	2	3	4	5
d_2 mm	80	160	240	320	400

Since 320 mm is closest to 330 mm, take the pitch diameter of worm gear $d_2 = 320$ mm

$\therefore$ Pitch diameter of worm $d_1 = 2a - d_2 = 2 \times 200 - 320 = 80$ mm

Lead angle $\gamma = \tan^{-1} \frac{m z_1}{d_1} = \frac{8 \times 4}{80} = 21.8°$

Number of starts on worm $z_1 = 4$

Number of teeth on worm wheel $z_2 = i z_1 = 10 \times 4 = 40$

Axial module $m = 8$mm

Normal module $m_n = m \cos\gamma = 8 \cos 21.8 = 7.428$ mm

Dimensions of worm

Number of starts on worm $z_1 = 4$

Pitch diameter of worm $d_1 = 80$mm

From Table 2.95 (PDHB)

Face length of worm $L_1 = (4.5 + 0.02\, z_1)\pi m$

$= (4.5 + 0.02 \times 2)\pi \times 8 \approx 114.1$ mm ≈ 115 mm

Depth of tooth $h_1 = 0.623\pi m = 0.623\pi \times 8 = 15.66$ mm

Addendum $h_{a_1} = 0.286\pi m = 0.286\pi \times 8 = 7.2$ mm

Outside diameter of worm $d_{a_1} = d_1 + 2h_a = 80 + 2 \times 7.2 = 94.4$ mm

Dedendum $h_{f_1} = (2.2\cos\gamma - 1)\,m = (2.2\cos 21.8 - 1)\,8 = 8.34$ mm

Root diameter of worm $d_{f_1} = d_1 - 2h_{f_1} = 80 - 2 \times 8.34 = 63.32$ mm

Diametral quotient $q = \dfrac{d_1}{m} = \dfrac{80}{8} = 10$

Dimensions of worm wheel

Number of teeth on worm wheel $z_2 = 40$

Pitch diameter of worm wheel $d_2 = 320$ mm

Face width of worm wheel $b = 2.15\pi m + 5$ ---- Table 2.95 (DDHB)

$= 2.15\pi \times 8 + 5 = 59$ mm ≈ 60 mm

Addendum $h_{a_2} = m\,[2\cos\gamma - 1] = 8\,[2\cos 21.8 - 1] = 6.86$ mm

Outside diameter of worm wheel $d_{a_2} = d_2 + 2h_{a_2}$

$= 320 + 2 \times 6.86 = 333.72$ mm

Dedendum $h_{f_2} = m\,[1 + 0.2\cos\gamma] = 8\,[1 + 0.2\cos 21.8] = 9.486$ mm

Root diameter of worm wheel $d_{f_2} = d_2 - 2h_{f_2} = 320 - 2 \times 9.486 = 301.028$ mm

Checking

From Table 2.98 (DDHB) for i = 10 and a = 200 mm

The selected worm gear designation is 4/40/10/8

i.e., $z_1 = 4$; $z_2 = 40$; $q = 10$; $m = 8$

Therefore the design is satisfactory.

ii) Input power capacity

From Lewis equation permissible load on weaker gear $F_{t_2} = \sigma_{02}\, m_n\, b\, Y\, C_v$

From Table 2.106 (DDHB) for phosphor bronze worm wheel

$\sigma_{02} = 55$ N/mm² [(σ_{02} is not equal to 103.5 N/mm²) Printing mistake in DDHB]

$Y = \pi y_2$

Approximate $y_2 = 0.17 - \dfrac{0.95}{40} = 0.14625$

Mean pitch line velocity of worm wheel $v_m = \dfrac{\pi d_2 n_2}{60000} = \dfrac{\pi \times 320 \times 200}{60000} = 3.351$ m/sec

Considering dynamic effect, velocity factor $C_v = \dfrac{6}{6 + v_m} = \dfrac{6}{6 + 3.351} = 0.64164$

$\therefore\ F_{t_2} = (55)(7.428)(60)(\pi \times 0.14625)(0.64164) = 7226.42$ N

Dynamic load $F_d = \dfrac{F_{t_2}}{C_v} = \dfrac{7226.42}{0.64164} = 11262.42$ N

Wear load $F_w = d_2\, bK$ ---- 2.557 (DDHB)

From Table 2.111 (DDHB) for $\gamma = 21.8°$ and hardened steel worm-phosphor bronze worm wheel

$$\text{Load stress factor } K = 0.69 \text{ N/mm}^2$$

$$\therefore F_w = (320)(60)(0.69) = 13248 \text{ N}$$

As the dynamic load does not exceed the allowable wear load, the safe transmitted load governs the design.

$\therefore$ **Input power capacity based on strength**

$$P = N = \frac{F_{t_2} \cdot n_2 r_2}{9550 \times 1000} = \frac{7226.42 \times 200 \times \left(\frac{320}{2}\right)}{9550 \times 1000} = 24.2 \text{ kW}$$

According to AGMA input power rating based on wear $P = N = \frac{n_1}{i} KQC_v, \text{kW}$

$$\text{where } Q = \frac{i}{i+2.5} = \frac{10}{10+2.5} = 0.8$$

$$C_v = \text{Velocity factor based on worm speed}$$

$$= \frac{2.3}{2.3 + v_w + \frac{3v_w}{i}}$$

$$\text{Velocity of worm } v_w = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 80 \times 2000}{60000} = 8.3776 \text{ m/sec}$$

$$\therefore C_v = \frac{2.3}{2.3 + 8.3776 + \frac{3 \times 8.3776}{10}} = 0.174363$$

$$K = \text{Pressure constant} = 0.485 \text{ kW/rpm for 200 mm centre distance}$$

$$\therefore N = \left(\frac{2000}{10}\right)(0.485)(0.8)(0.174363) = 13.53 \text{ kW}$$

According to AGMA input power rating based on heat dissipation $P = N = \frac{3650 a^{1.7}}{i+5}$

$$\text{where } a = \text{Centre distance in meters}$$

$$\text{i.e., } a = 200 \text{ mm} = 0.2 \text{ m}$$

$$\therefore P = N = \frac{3650(0.2)^{1.7}}{(10+5)} = 15.7744 \text{ kW}$$

The least among the above three values will be the permissible power

$\therefore$ Input power capacity $N = 13.53$ kW. It is based on AGMA wear recommendations.

iii) Efficiency

Considering worm as the driver

$$\eta = \frac{\tan\gamma\left[\cos\alpha_n \cos\gamma - \mu \sin\gamma\right]}{\cos\alpha_n \sin\gamma + \mu\cos\gamma} \quad \text{---- 2.587 (DDHB)}$$

$$v_r = \text{Rubbing velocity} = \frac{\pi d_1 n_1}{60000 \cos\gamma} \quad \text{---- 2.544 a (DDHB)}$$

$$= \frac{\pi \times 80 \times 2000}{60000 \times \cos 21.8} = 9.023 \text{ m/sec}$$

$$\mu = 0.025 + \frac{V_r}{305} \; (\because 2.75 < V_r < 20 \text{ m/sec}) \quad \text{---- 2.543 b (DDHB)}$$

$$= 0.025 + \frac{9.023}{305} = 0.054583$$

$$\therefore \eta = \frac{\tan 21.8[(\cos 20)(\cos 21.8) - (0.054583)(\sin 21.8)]}{(\cos 20)(\sin 21.8) + (0.054583)(\cos 21.8)}$$

$$= 0.8522 = 85.22\%$$

or

$$\eta = \frac{\tan\gamma}{\tan(\gamma + \rho)} \quad \text{---- 2.566 a (DDHB)}$$

Where $\rho = \text{Friction angle} = \tan^{-1}\mu = \tan^{-1} 0.054583 = 3.1243°$

$$\therefore \eta = \frac{\tan 21.8}{\tan(21.8 + 3.1243)} = 0.86 = 86\%$$

iv) Heat Balance

a) Heat generated $H_g = \frac{\mu F_n V_r}{1000}$ kW ---- 2.576 a (DDHB)

Where F_n = Normal force $= \frac{2M_{t_2}}{d_2 \cos\gamma \cos\alpha} = \frac{F_{t_2}}{\cos\gamma \cos\alpha}$ ---- 2.546 (DDHB)

$$= \frac{7226.42}{(\cos 21.8)(\cos 20)} = 8282.5 \text{ N}$$

$$\therefore H_g = \frac{(0.054583)(8282.5)(9.023)}{1000} = 4.08 \text{ kW}$$

b) Heat dissipated $H_d = \frac{h_{cr} A (t_g - t_a)}{1000}$ kW ---- 2.577 (DDHB)

Where A = Radiating area of housing in $m^2 = 14.4\, a^{1.7}$

$$= 14.4 (0.2)^{1.7} = 0.9335 \text{ m}^2$$

t_g = Temperature of gear = 65° C (assume) = (273 + 65) = 338° K

t_a = Ambient temperature = 25° C (assume) = 273 + 25 = 298° K

From figure 2.73 (DDHB) for A = 0.9335 m^2

Coefficient of heat transfer $h_{cr} = 320 \text{ W/m}^2\text{K}$

$$\therefore H_d = \frac{(320)(0.9335)(40)}{1000} = 11.95 \text{ kW}$$

As heat generated is less than heat dissipation capacity, artificial cooling arrangement is not necessary.

Example 5.10

Design a worm gear drive to transmit a power of 2 kW at 1000 rpm. The speed ratio is 20 and the centre distance is 200 mm.

Data :

$$P = N = 2\text{ kW};\ n_1 = 1000\text{ rpm};\ i = 20;\ a = 200\text{ mm}$$

Solution :

i) Dimension of worm and worm gear

Velocity ratio $i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{\pi d_2}{p_z}$

Speed of worm gear $n_2 = \frac{n_1}{i} = \frac{1000}{20} = 50\text{ rpm}$

Pitch diameter of worm $d_1 = \frac{a^{0.875}}{3.5}$ ----2.519 b (DDHB)

where a = Centre distance in meters = 0.2 m

$\therefore\ d_1 = \frac{(0.2)^{0.875}}{3.5} = 0.069876\text{ m} = 69.876\text{ mm} \approx 70\text{ mm}$

Centre distance $a = \frac{d_1 + d_2}{2}$

i.e., $200 = \frac{70 + d_2}{2}$

$\therefore$ Pitch diameter of worm gear $d_2 = 330$ mm, based on the first estimate

Also $d_1 \approx 3\,p_c$

i.e., $70 = 3\pi m$

$\therefore$ Axial module m = 7.42 mm = 8 mm [select standard module from Table 2.3]

Now $i = \frac{\pi d_2}{p_z} = \frac{\pi d_2}{\pi m_x z_1} = \frac{d_2}{m z_1}$

$\therefore\ d_2 = imz_1 = 20 \times 8 \times z_1 = 160\,z_1$

Thus for various values of z_1 the value of d_2 is tabulated as below

Z_1	1	2	3	4
d_2 mm	160	320	480	640

Since 320 mm is closest to 330 mm, take the pitch circle diameter of worm gear $d_2 = 320$ mm

$\therefore$ Pitch diameter of worm $d_1 = 2a - d_2 = 2 \times 200 - 320 = 80$ mm

Number of starts on worm $z_1 = 2$

Number of teeth on worm wheel $z_2 = iz_1 = 20 \times 2 = 40$

Lead angle $\gamma = \tan^{-1} \dfrac{mz_1}{d_1} = \dfrac{8 \times 2}{80} = 11.31°$

Assume pressure angle α = 20° full depth involute.

Axial module m = 8mm

Normal module $m_n = m\cos\gamma = 8 \times \cos 11.31 = 7.8446$ mm

Dimensions of worm

Number of starts on worm $z_1 = 2$

Pitch diameter of worm d_1 = 80mm

From Table 2.95 (DDHB)

Face length of worm $L_1 = (4.5 + 0.02\,z_1)\pi m$

$= (4.5 + 0.02 \times 2)\pi \times 8 = 114.1$ mm ≈ 115 mm

Depth of tooth $h_t = 0.686\pi m = 0.686 \times \pi \times 8 = 17.24$ mm

Addendum $h_{a_1} = 0.318\pi m = 1\,m = 1 \times 8 = 8$ mm

Outside diameter of worm $d_{a_1} = d_1 + 2\,h_{a_1} = 80 + 2 \times 8 = 96$ mm

Dedendum $h_{f_1} = (2.2\cos\gamma - 1)\,m = (2.2\cos 11.31 - 1)\,8 = 9.26$ mm

Root diameter of worm $d_{f_1} = d_1 - 2\,h_{f_1} = 80 - 2 \times 9.26 = 61.48$ mm

Diametral quotient $q = \dfrac{d_1}{m} = \dfrac{80}{8} = 10$

Dimensions of worm wheel

Number of teeth on worm wheel $z_2 = 40$

Pitch circle diameter of worm wheel $d_2 = 320$ mm

From Table 2.95 (DDHB)

Face width of worm wheel $b = 2.38\pi m + 6.25 = 66.1$ mm

From Table 2.103 (DDHB)

$b \le 0.75\,d_1$; i.e., $b \le 0.75 \times 80$ $\therefore$ $b \le 60$ mm

$\therefore$ Take face width b = 60mm

Addendum $h_{a_2} = m\,(2\cos\gamma - 1) = 8\,[2\cos 11.31 - 1] = 7.7$ mm

Outside diameter of worm wheel $d_{a_2} = d_2 + 2\,h_{a_2} = 320 + 2 \times 7.7 = 335.4$ mm

Dedendum $h_{f_2} = m\,[1 + 0.2\cos\gamma] = 8\,[1 + 0.2\cos 11.31] = 9.6$ mm

Root diameter of worm wheel $d_{f_2} = d_2 - 2\,h_{f_2} = 320 - 2 \times 9.6 = 300.8$ mm

Checking

From Table 2.98 (DDHB) for $i = 20$ and centre distance $a = 200$ mm.

The selected worm gear designation is 2/40/10/8

i.e., $z_1 = 2$; $z_2 = 40$; $q = 10$; $m = 8$

Therefore the design is satisfactory.

ii) Check the gear for gear tooth strength

From Lewis equation permissible transmitted load $F_{t_2} = \sigma_{02}\, b\, Y\, m_n\, C_v$

Assume hardened steel worm and phosphor bronze worm wheel

$\therefore$ From Table 2.106 (DDHB) for phosphor bronze worm wheel $\sigma_{02} = 55\ N/mm^2$ [(σ_{02} is not equal to 103.5 N/mm^2) Printing mistake in DDHB]

$$\text{Approximate } y_2 = 0.154 - \frac{0.912}{40} = 0.1312$$

$$Y = \pi y_2 = \pi \times 0.1312 = 0.4122$$

Mean pitch line velocity of worm wheel $v_m = \dfrac{\pi d_2 n_2}{60000} = \dfrac{\pi \times 320 \times 50}{60000} = 0.838$ m/sec

Considering dynamic effect, velocity factor $C_v = \dfrac{6}{6 + v_m} = \dfrac{6}{6 + 0.838} = 0.8775$

$\therefore$ Permissible transmitted load $F_{t_2} = (55)(60)(0.4122)(7.8446)(0.8775) = 9363.5$ N

$$\text{Transmitted load } F_{t_2} = \frac{9550 \times 1000 \times N}{n_2 r_2} = \frac{9550 \times 1000 \times 2}{50 \times \left(\frac{320}{2}\right)} = 2387.5\ N$$

Estimated dynamic load $F_d = \dfrac{F_t}{C_v} = \dfrac{2387.5}{0.8775} = 2720.8$ N

$$\text{Allowable wear load } F_w = d_2 bK$$

From Table 2.111 (DDHB) for hardened steel worm and phosphor bronze worm wheel and $\gamma = 11.31°$

Load stress factor $K = 0.69\ MPa = 0.69\ N/mm^2$ $\therefore$ $F_w = (320)(60)(0.69) = 13248$ N

As the allowable wear load is greater than the estimated dynamic load, and the allowable transmitted load is greater than the required transmitted load, the design is satisfactory from stand point of strength and wear of the gear teeth. In fact the permissible power is 7.844 kW.

$$\text{i.e., } 9363.5 = \frac{9550 \times 1000 \times N}{(50)\left(\frac{320}{2}\right)}$$

$$\therefore \text{ Permissible power } N = 7.844\ kW$$

iii) Efficiency

Considering worm as the driver

$$\eta = \frac{\tan\gamma\left[\cos\alpha_n \cos\gamma - \mu \sin\gamma\right]}{\cos\alpha_n \sin\gamma + \mu \cos\gamma} \qquad \text{---- 2.587 (DDHB)}$$

$$v_r = \text{Rubbing velocity} = \frac{\pi d_1 n_1}{60000 \cos\gamma} \qquad \text{---- 2.544 a (DDHB)}$$

$$= \frac{\pi \times 80 \times 1000}{60000 \cos 11.31} = 4.272\ m/sec$$

$$\mu = 0.025 + \frac{v_r}{305} \text{ since } 2.75 < v_r < 20\ m/sec \qquad \text{---- 2.543 b (DDHB)}$$

$$= 0.025 + \frac{4.272}{305} = 0.039$$

$$\therefore \ \eta = \frac{\tan 11.31[\cos 20 \cos 11.31 - 0.039 \sin 11.31]}{\cos 20 \sin 11.31 + 0.039 \cos 11.31} = 0.8212 = 82.12\%$$

iv) Heat balance

a) Heat generated $H_g = \frac{\mu F_n v_r}{1000}$ kW ---2.576 a (DDHB)

$$F_n = \text{Normal force} = \frac{2Mt_2}{d_2 \cos\gamma \cos\alpha} = \frac{F_{t_2}}{\cos\gamma \cos\alpha} \quad \text{----2.546 (DDHB)}$$

$\therefore$ Permissible normal force $F_n = \frac{9363.5}{\cos 11.31 \cos 20} = 10161.8$ N

$\therefore$ Permissible heat generated $H_g = \frac{(0.039)(10161.8)(4.272)}{1000} = 1.693$ kW

b) Heat dissipated $H_d = \frac{h_{cr} A (t_g - t_a)}{1000}$ kW ----5.77 (DDHB)

A = Radiating area of housing in $m^2 = 14.4\, a^{1.7}$

$= 14.4\,(0.2)^{1.7} = 0.9335\ m^2$

t_g = Temperature of gear = 65° C (assume) = 273 + 65 = 338° K

t_a = Ambient temperature = 25° C (assume) = 273 + 25 = 298° K

$\therefore$ Temperature difference = $t_g - t_a = 338 - 298 = 40°$ K

From Fig. 2.73 (DDHB) for $A = 0.9335\ m^2$

Coefficient of heat transfer h_{cr} = 320 W/m² K

$\therefore$ Heat dissipated $H_d = \frac{(320)(0.9335)(40)}{1000} = 11.95$ kW

Since heat generated is less than heat dissipation capacity, artificial cooling arrangement is not necessary.

Example 5.11

The following data refer to a worm and worm gear drive

a) Centre distance 200 mm

b) Pitch circle diameter of the worm 80 mm

c) Number of start 4

d) Axial module 8 mm

e) Transmission ratio 20

f) The worm gear is made of phosphor bronze with an allowable bending stress of 55 MPa

g) The worm is made of hardened and ground steel

h) Tooth form is 20° full depth involute.

Determine the following :

a) Number of teeth on the worm gear

b) Lead angle

c) Face width of worm gear to transmit 15 kW of power at 1750 rpm of the worm based on the beam strength of the worm gear **VTU, July/August 2004**

Data :

$$a = 200\text{ mm};\ d_1 = 80\text{ mm};\ z_1 = 4;\ m = 8\text{ mm};\ i = 20;\ \sigma_{02} = 55\text{ N/mm}^2;\ \alpha = 20^\circ\text{ FD};$$
$$P = N = 15\text{ kW};\ n_1 = 1750\text{ rpm}$$

Solution :

a) Number of teeth on the worm gear

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{\pi d_2}{p_z}$$

∴ Number of teeth on the worm gear $z_2 = iz_1 = 20 \times 4 = 80$

b) Lead angle

$$\text{Lead angle } \gamma = \tan^{-1}\frac{mz_1}{d_1} = \tan^{-1}\frac{8 \times 4}{80} = 21.8^\circ$$

c) Face width of worm gear

$$\text{Transmitted load } F_{t_2} = \frac{9550 \times 1000 N}{n_2 r_2} = \frac{9550 \times 1000 \times N}{n_2 r_2}$$

$$\text{Speed of worm gear } n_2 = \frac{n_1}{i} = \frac{1750}{20} = 87.5\text{ rpm}$$

$$\text{Centre distance } a = \frac{d_1 + d_2}{2}$$

$$\text{i.e., } 200 = \frac{80 + d_2}{2}$$

∴ Pitch circle diameter of worm gear $d_2 = 320$ mm

$$\text{Pitch circle radius of worm gear } r_2 = \frac{d_2}{2} = \frac{320}{2} = 160\text{ mm.}$$

$$\therefore F_{t_2} = \frac{9550 \times 1000 \times 15}{87.5 \times 160} = 10232.143\text{ N}$$

From Lewis equation permissible transmitted load

$$F_{t_2} = \sigma_{02}\, b Y m_n C_v$$

$$\text{Approximate } y_2 = 0.154 - \frac{0.912}{80} = 0.1426$$

$$\therefore Y = \pi y_2 = \pi \times 0.1426 = 0.448$$

$$\text{Normal module } m_n = m\cos\gamma = 8 \times \cos 21.8 = 7.428\text{ mm}$$

$$\text{Mean pitch line velocity of worm wheel } v_m = \frac{\pi d_2 n_2}{60000}$$

$$= \frac{\pi \times 320 \times 87.5}{60000} = 1.466\text{ m/sec}$$

Considering the dynamic effect, velocity factor $C_v = \frac{6}{6+v_m} = \frac{6}{6+1.466} = 0.803635$

$\therefore$ 10232.143 = (55) (b) (0.448) (7.428) (0.803635)

ie., b = 69.565 mm

$\therefore$ Face width b = 70mm.

Example 5.12

Design a worm gear drive to transmit 40 kW at 500 rpm of worm. The ratio is 25. Material for the gear is phosphor bronze and that of worm is hardened steel. Determine the efficiency of the drive also.

VTU, August 2001

Data :

P = N = 40 kW; n_1 = 500 rpm; i = 25; Worm material - Hardened steel; Worm gear material - Phosphor bronze.

Solution :

Choose centre distance which satisfies the given condition by trial and error. Now the problem is similar to Example : 5.10.

5.18 WORM GEAR FORCES

In the analysis of worm gear tooth forces, it is assumed that the worm is the driving member while the worm wheel is the driven member. The three components of the gear tooth forces between the worm and the worm wheel are shown in Fig. 5.15 a and 5.15 b. The components of the resultant force acting on the worm are as follows.

F_{t_1} = Tangential component of worm

F_{a_1} = Axial component of worm

F_{r_1} = Radial component of worm

The components F_{t_2}, F_{a_2} and F_{r_2}, acting on the worm wheel are defined in a similar way. The force acting on the worm wheel is equal and opposite reaction of the force acting on the worm . The direction of the three components depends upon three factors, they are

i) Driving element i.e., worm or worm wheel

ii) Threads on the worm i.e. right hand or left hand thread

iii) Rotation of worm i.e., clockwise or anticlockwise

Therefore in Fig. 5.15 a, the direction of the three components are decided by making the following assumptions ;

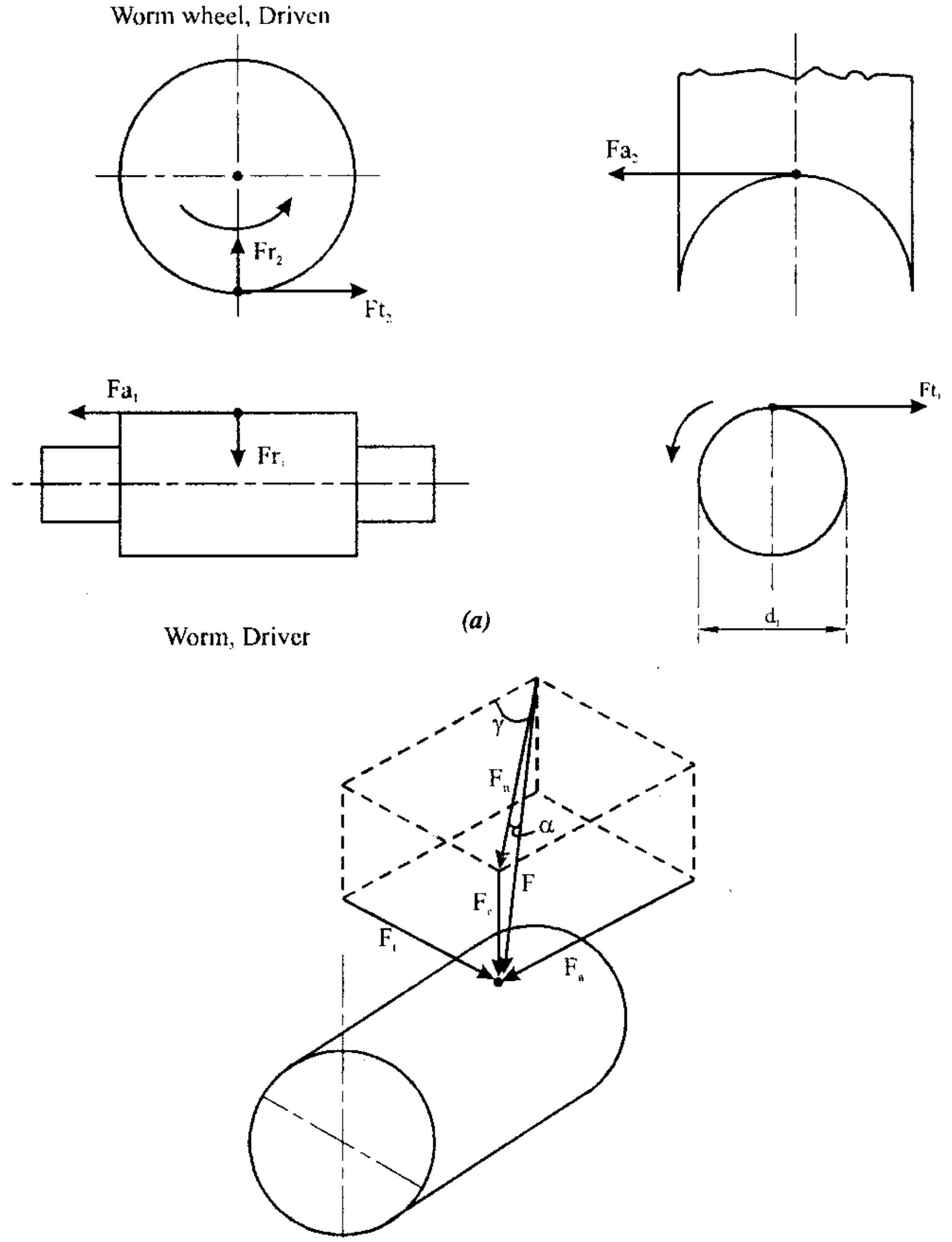
i) Worm is the driving element

ii) Worm has right handed threads and

iii) Worm rotates in anticlockwise direction when seen from left side.

In order to decide the rotation of worm wheel, the worm is considered as screw and worm wheel as nut. Applying right hand thumb rule (right handed threads for worm) and keeping the fingers in the direction of rotation of worm, the thumb pointing to the left indicates the motion of the screw. The nut will move in the opposite direction i.e., to the right. Therefore, the worm wheel rotates in the counter clockwise direction as shown in the Fig. 5.15 a.

The following are the magnitude of the forces.



(b) *Fig 5.15*

$$F_{t_1} = \frac{M_{t_1}}{r_1}$$

$$F_{a_1} = F_{t_1}\left[\frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu\cos\gamma}\right]$$

$$F_{r_1} = F_{t_1} \times \frac{\sin\alpha_n}{[\cos\alpha_n \sin\gamma + \mu\cos\gamma]}$$

Referring the Fig. 5.15(a)

$$F_{t_2} = F_{a_1}$$
$$F_{a_2} = F_{t_1}$$
$$F_{r_2} = F_{r_1}$$

where

M_{t_1} = Torque on the worm shaft

r_1 = Radius of worm = $\frac{d_1}{2}$

α_n = Normal pressure angle.

d_1 = Pitch circle diameter of worm

d_2 = Pitch circle diameter of worm wheel

m = Module

q = Diametral quotient = $\frac{d_1}{m}$

z_1 = Number of starts on the worm

z_2 = Number of teeth on worm wheel

γ = Lead angle of worm = $\tan^{-1}\left(\frac{z_1}{q}\right) = \tan^{-1}\left(\frac{l}{\pi d_1}\right)$

$= \tan^{-1}\left(\frac{p_z}{\pi d_1}\right)$

$p_z = l$ = Lead = Number of starts × pitch.

μ = Coefficient of friction

Designation of worm $z_1/z_2/q/m$

Example 5.13

A pair of worm and worm wheel is designated as 3/60/10/6. The worm is transmitting 2.5 kW power at 1440 r.p.m to the worm wheel. The coefficient of friction is 0.1 and the normal pressure angle is 20°. Determine the components of gear tooth force acting on the worm and the worm wheel.

Data :

Designation of worm 3/60/10/6

i.e., Number of starts on worm $z_1 = 3$

Number of teeth on worm wheel $z_2 = 6$

Diametral quotient $q = 10$

Module $m = 6$

$P = N = 2.5\,KW$

$n_1 = 1440$ rpm.

$\mu = 0.1$

$\alpha_n = 20°$

Solution :

$$q = \frac{d_1}{m}$$

$$\therefore \quad 10 = \frac{d_1}{6}$$

$\therefore$ Pitch circle diameter of worm $d_1 = 60\,mm$

$$\therefore \quad r_1 = \frac{60}{2} = 30\,mm$$

Torque on the worm shaft $M_{t_1} = \dfrac{60 \times 10^6 \times N}{2\pi n_1} = \dfrac{60 \times 10^6 \times 2.5}{2\pi \times 1440} = 16578.64\,Nmm$

Lead angle of worm $\gamma = \tan^{-1}\left(\dfrac{z_1}{q}\right) = \tan^{-1}\left(\dfrac{3}{10}\right) = 16.7°$

$\therefore$ Tangential force on worm $F_{t_1} = \dfrac{M_{t_1}}{r_1} = \dfrac{16578.64}{30} = 552.62\,N$

Axial force on worm $F_{a_1} = F_{t_1} \times \left[\dfrac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu\cos\gamma}\right]$

$$= 552.62 \times \left[\frac{(\cos 20)(\cos 16.7) - (0.1)(\sin 16.7)}{(\cos 20)(\cos 16.7) + (0.1)(\cos 16.7)}\right] = 1316.25\,N$$

Radial force on worm $F_{r_1} = F_{t_1} \times \dfrac{\sin\alpha_n}{[\cos\alpha_n \sin\gamma + \mu\cos\gamma]}$

$$= 552.62 \times \frac{\sin 20}{(\cos 20)(\sin 16.7) + (0.1)(\cos 16.7)} = 516.677\,N$$

The force components acting on the worm wheel are as follows,

Tangential force on worm wheel $F_{t_2} = F_{a_1} = 1316.25\,N$

Axial force on worm wheel $F_{a_2} = F_{t_1} = 552.62\,N$

Radial force on worm wheel $F_{r_2} = F_{r_1} = 516.677\,N$

Example 5.14

A two teeth right hand worm transmits 2 kW at 1500 rpm to a 36 teeth wheel. The module of the wheel is 5 mm and the pitch diameter of the worm is 60 mm. The normal pressure angle is 14.5°. The coefficient of friction is found to be 0.06.

i) Find the centre distance, the lead and lead angle

ii) Determine the forces.

iii) Determine the efficiency of the drive **VTU, July/August 2002**

Data :

$$P = N = 2\,kW;\ n_1 = 1500\,rpm;\ z_1 = 2;\ z_2 = 36;\ m = 5\,mm;\ d_2 = 60\,mm;$$
$$\alpha_n = 14½°;\ \mu = 0.06$$

Solution :

i) Centre distance, lead and lead angle

Pitch circle diameter of worm $d_1 = 3\pi m = 3 \times \pi \times 5 = 47.12\ mm \approx 48\ mm$

$$\therefore \text{ Centre distance } a = \frac{d_1 + d_2}{2} = \frac{48+60}{2} = 54\ mm$$

$$\text{Lead } p_z = p_x z_1 = \pi m z_1 = \pi \times 5 \times 2 = 31.416\ mm$$

$$\text{Lead angle } \gamma = \tan^{-1}\frac{p_z}{\pi d_1} = \tan^{-1}\frac{31.416}{\pi \times 48} = 11.768°$$

ii) Forces

$$\text{Tangential force on worm } F_{t_1} = \frac{9550 \times 1000 \times N}{n_1 r_1}$$

$$= \frac{9550 \times 1000 \times 2}{1500 \times \left(\frac{48}{2}\right)} = 530.56\ N$$

$$\text{Axial force on worm } F_{a_1} = F_{t_1}\left(\frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu\cos\gamma}\right)$$

$$= 530.56\left[\frac{(\cos 14½)(\cos 11.768) - (0.06)(\sin 11.768)}{(\cos 14½)(\sin 11.768) + (0.06)(\cos 11.768)}\right]$$

$$= 1937.44\ N$$

$$\text{Radial force on worm } F_{r_1} = F_{t_1}\left[\frac{\sin\alpha_n}{\cos\alpha_n \sin\gamma + \mu\cos\gamma}\right]$$

$$= 530.56\left[\frac{\sin 14½}{(\cos 14½)(\sin 11.768) + (0.06)(\cos 11.768)}\right]$$

$$= 518.52\ N$$

The force components acting on the worm wheel are as follows.

Tangential force on worm wheel $F_{t_2} = F_{a_1} = 1937.44\ N$

Axial force on worm wheel $F_{a_2} = F_{t_1} = 530.56\,N$

Radial force on worm wheel $F_{r_2} = F_{r_1} = 518.52\,N$

iii) Efficiency

Since worm is the driver

$$\eta = \tan\gamma\left[\frac{\cos\alpha_n \cos\gamma - \mu\sin\gamma}{\cos\alpha_n \sin\gamma + \mu\cos\gamma}\right] \qquad \text{----2.587 (DDHB)}$$

$$= \tan 11.768\left[\frac{(\cos 14\tfrac{1}{2})(\cos 11.768) - (0.06)(\sin 11.768)}{(\cos 14\tfrac{1}{2})(\sin 11.768) + (0.06)(\cos 11.768)}\right]$$

$$= 0.76075 = 76.075\%$$

or

$$\eta = \frac{\tan\gamma}{\tan(\gamma+\rho)} \qquad \text{----2.566 a (DDHB)}$$

$$\rho = \text{Friction angle} = \tan^{-1}\mu = \tan^{-1}(0.06) = 3.43363$$

$$\therefore\ \eta = \frac{\tan 11.768}{\tan(11.768 + 3.43363)} = 0.7667 = 76.67\%$$

REVIEW QUESTIONS

1. Explain briefly the formative number of teeth of bevel gears.
VTU, February 2002, July/August 2002
2. Explain formative number of teeth. **VTU, August 2001**
3. Explain "Self locking effect" in case of a worm gear drive. **VTU, August 2001**
4. Explain spiral bevel gear and hypoid gears? **BU, August/September 2001**
5. Sketch and describe a meshing worm gear and worm wheel. What are their advantages?
BU, August/September 2001

EXERCISES

1. A pair of bevel gears is to be designed to transmit a power of 15 kW from a shaft running at 750 rpm to another perpendicular shaft to be run at 350 rpm.
2. Design a worm and worm wheel to transmit a power of 10 kW with a speed reduction ratio of 20 and a centre distance of 220 mm. The worm speed is 1000 rpm.
3. Design a worm gear reducer to transmit 7.5 kW at 100 rpm. The input speed from the motor is 1500 rpm. Calculate the efficiency of the drive.
4. Design a bevel gear drive for a vertical drilling machine in which speed is reduced from 1500 rpm to 750 rpm. The motor power rating is 5 kW. Machine operates for 8 hours/day and steady load condition prevails. Pinion and gear and made of steel with allowable static stress of 200 MPa and 235 MPa respectively. Check the design for dynamic and wear strength.
5. A vertical drilling machine requires a set of bevel gears to reduce the speed from 2500 rpm to 1250 rpm at the spindle. Design the bevel gear drive with 20° full depth tooth form. The power to be transmitted is 20 kW select proper materials for compact design and check for dynamic and wear loads.
6. Design a pair of right-angled bevel gears to transmit a power of 15 kW from a shaft running at a speed of 750 rpm to a perpendicular shaft to be run at 250 rpm. Suggest suitable surface hardness for the gear pair. **VTU, Jan/Feb. 2005**
7. Design a pair of bevel gears to transmit a power of 25 kW from a shaft rotating at 1200 rpm to a perpendicular shaft to be rotated at 400 rpm. **VTU, Jan/Feb. 2006**
8. Determine the cone pitch angles, pitch diameters for the following bevel gear pairs:
   i) For shaft angle 77° (Acute angle bevel gearing)
   ii) For shaft angle 147° (Obtuse angle bevel gearing)

   The module is 5 mm and the number of teeth on the pinion and the gear are 14 and 42, respectively. Draw the sketches of gearing. **VTU, July 2006**

9. a) A pair of straight bevel gears transmits 15kW at 1250 rpm of 120mm diameter pinion. The speed reduction is 3.5. Use 14.5° involute tooth system. The angle between the shaft axes is 90°. The pinion is made of case hardened alloy steel with allowable static stress 343.34 MPa and gear is cast steel of 0.20%C heat treated with allowable static stress 191.295 MPa. Determine module, face width, number of teeth on pinion and gear. Suggest suitable surface hardness for the gear pair. Take the service factor as 1.5 and assume the teeth are generated.

   b) Explain formative number of teeth in bevel gears. **VTU, Dec. 06/Jan. 2007**

10. A pair of 20° full depth involvte teeth bevel gears connect two shafts at right angles having velocity ratio 3:1. The gear is made of cast steel, 0.20% untreated and the pinion material is of steel, C 30 heat-treated. The pinion has 20 number of teeth and transmits 40 kW at 750 rpm. Determine: i) Module ii) Face width and iii) Pitch diameters. Assume width of gear face as one third of the length of pitch cone. **VTU, July. 2007**

11. Design a worm gear drive for a speed of 500 rpm of the worm transmitting 20 kW. The velocity ratio is 25:1. The material of the gear is phosphorus bronze and the worm is hardened steel. Determine the efficiency of the drive also. **VTU, Dec. 07/Jan. 2008**

12. A pair of straight tooth bevel gears at right angles is to transmit 5 kW at 1200 rpm of the pinion. The diameter of the pinion is 80mm and the velocity ratio is 3.5. The tooth form is 14½. Both the pinion and gear are cast iron with allowable stress of 55 MN/m^2. Determine the modue, face width from the stand point of strength and also check the design from the stand point of dynamic load and wear load. **VTU, Jun/July 2008**

7. a) A hardened steel worm rotating at 1250 rpm transmits power to a phosphor bronze gear with transmission ratio 15:1. The centre distance of drive is 225 mm. Design the gear drive, and estimate the power rating of the drive from stand point of strength and heat dissipation. The teeth are 14½° full depth involute form.

   b) Explain the meaning of " Formative number of teeth" as referred to bevel gears.

VTU, Dec. 08/Jan. 2009